

Namaste



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Jonathan J. Crabtree

Elementary Mathematics Historian

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PART 1. Negative & Positive Quantities on a Brahmaguptan Plane for India's Primary Classes



**PART 2. Why India Must Change its Story
Bad Maths History \Rightarrow Sad Maths Mystery**

PART 1. Negative & Positive Quantities on a Brahmaguptan Plane for India's Primary Classes



Jonathan J. Crabtree, Elementary Mathematics Historian, Australia.

9th National conference: Technology & Innovations in Math Education

Biennial Conference of the Mathematics Department of the IIT Bombay.

Jointly organized by IISER, Pune & BATU, Lonere.

Pune India, 27th December 2019

“There are no studies demonstrating how negative numbers and algebra can be taught to such students in a meaningful way.”



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Mathematics Education in India *Status and Outlook*

Edited by R. Ramanujam and K. Subramaniam

Homi Bhabha Centre for Science Education

Tata Institute of Fundamental Research

“Negative numbers, usually introduced early in class 6, are known to be a problem area.”



Mathematics Education in India *Status and Outlook*

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“Negative times negative is positive is problematic to justify.”



**A quote I jotted down from Dr. Shailesh Shirali
this morning, 27 December.**

“Negative times negative is positive is problematic to justify.”



Now, Brahmagupta's simple definition of zero lets children understand $-1 \times -1 = +1$ with absolute rock-solid certainty and intuitive conviction!

***“Something is amiss. Why are we adrift?
What should be done?
We need to gain insight from history.”***



Some quotes I jotted down from Professor Dinesh Singh's inaugural TIME2019 address 26 December.

“We must offer something tangible to policy holders. Sanskrit is in our DNA, but we never bring it into the learning of mathematics”



Some quotes I jotted down from Professor Dinesh Singh's inaugural TIME2019 address 26 December.

“We must offer something tangible to policy holders. Sanskrit is in our DNA, but we never bring it into the learning of mathematics”



My goal is to answer questions like these and to give India's leaders and teachers the solutions children need.

“We must offer something tangible to policy holders. Sanskrit is in our DNA, but we never bring it into the learning of mathematics”



To solve India’s primary-level mathematics education problems, I am creating a free ‘Sanskrit-based’ alternative to Arithmetic called Podometric.

“Please forget everything you learned in school, because you have not learned it.”

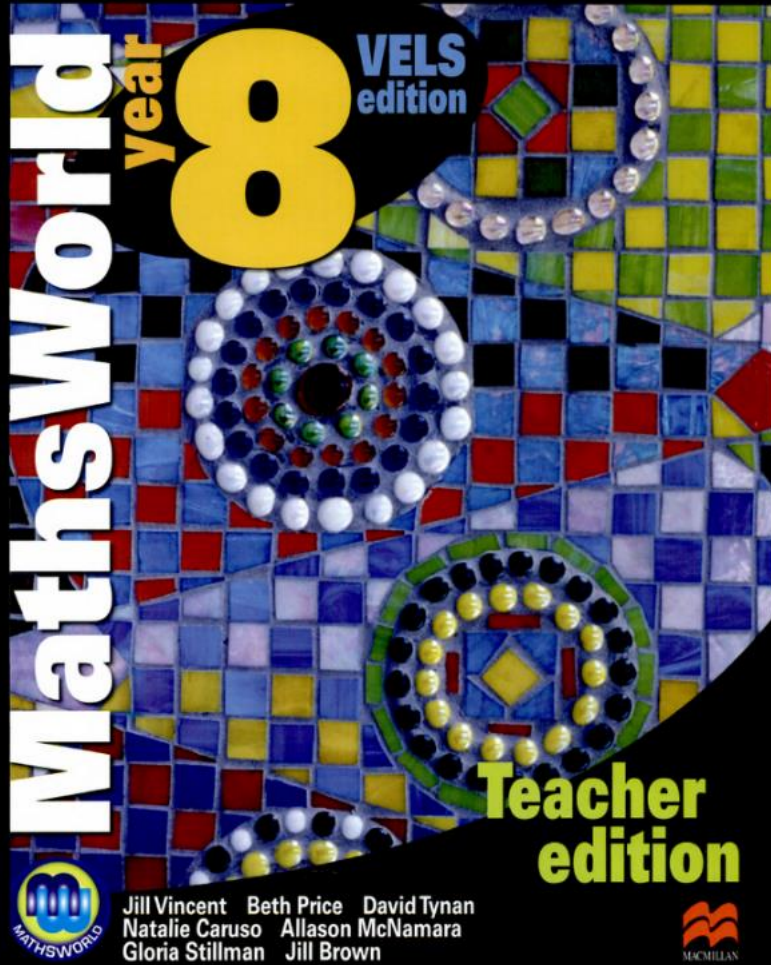


Edmund Landau, *Foundations of Analysis*

Please **forget** much about what you learned
in school about **arithmetic**, because you
don't understand my definition of **zero**.



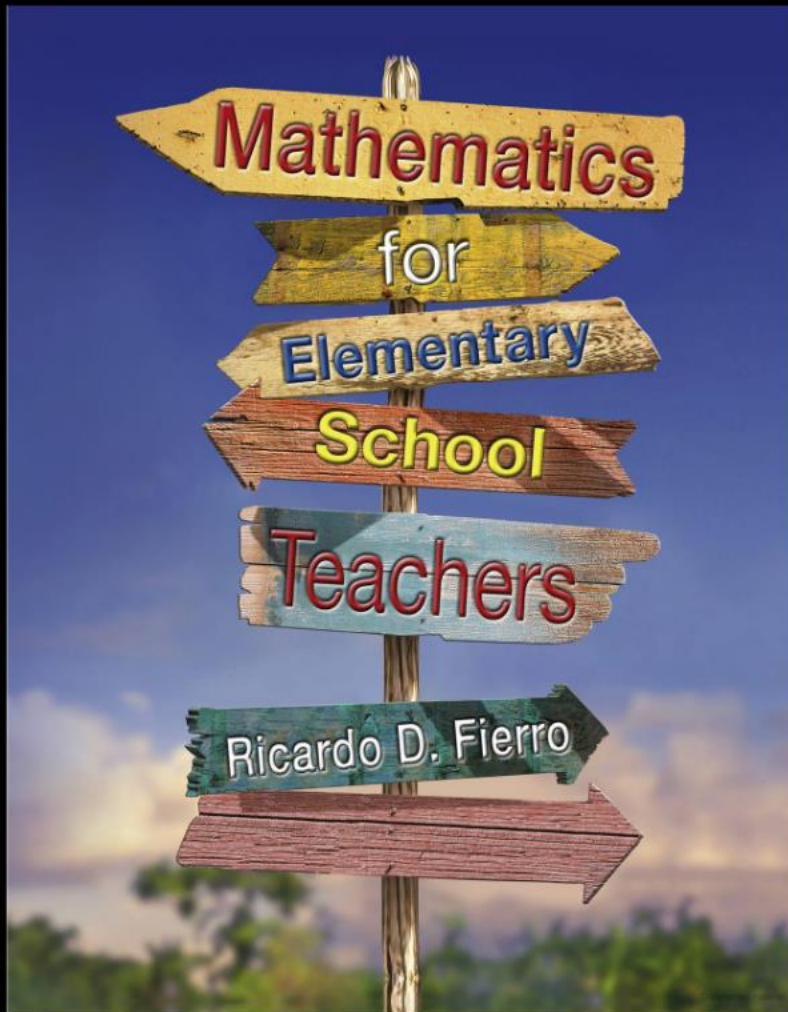
What Brahmagupta might say if he were alive today.



“Brahmagupta... defined zero as the result of subtraction of a number from itself.”

“He also gave the following rules for operations on what he called ‘fortunes’ (positive numbers) and ‘debts’ (negative numbers).”

“The product... of two debts is one fortune.”



“Brahmagupta (598 – 670 CE) was a Hindu mathematician and astronomer who lived in the first century.”

“He used negative integers to represent debts and positive integers to represent assets.”

“The product ... of two debts is one fortune.”

Brahmagupta

"The product of a positive and a negative (number) is negative; of two negatives is positive; positive multiplied by 'positive is positive."

Mahāvīra

"In the multiplication of two negative or two positive numbers the result is positive; but it is negative in the case of (the multiplication of) a positive and a negative number."

Śrīpati

"On multiplying two negative or two positive numbers (the product is) positive; in the multiplication of positive and negative (the result is) negative."

Bhāskara II

"The product of two positive or two negative (numbers) is positive; the product of positive and negative is negative." The same rule is stated by Nārāyaṇa.

SOURCE: *History of Hindu Mathematics: A Source Book, Part II, Algebra*, Bibhutibhusan Datta and Bidyāraṇya Avadesh Narayan Singh, pp. 22-23, Motilal Banarsidass, Lahore, 1938.

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The Nine Chapters on the Mathematical Art
(九章算术 Jiǔzhāng Suànshù circa 100 CE).

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The Nine Chapters on the Mathematical Art: Companion and Commentary,
Shen Kangshen, John N. Crossley and Anthony W. C. Lun, Oxford University Press, 2000.

If Maths is Language,
it must obey the
'Parts of Speech'
of language.

The Grammar of Arithmetic

Negative Seven
Minus
Negative Four?

Most adults answer
“Negative Eleven?”

Most adults answer
“Negative Eleven?”

Why?

Adjective Adjective
Negative Seven

Verb

Minus

Adjective Adjective
Negative Four?

Adjective Adjective
Negative Seven

Verb

Minus

Adjective Adjective
Negative Four?

No Nouns!

Adjective Adjective
Negative Seven

Verb

Minus

Adjective Adjective
Negative Four?

Adults struggle for rules memorized without
meaning *'is it two negatives make a plus?'*

Seven Negatives
Minus
Four Negatives?

Every child answers
“Three Negatives!”

Every child answers
“Three Negatives!”
Why?

Adjective Nouns
Seven Negatives
Verb
Minus
Adjective Nouns
Four Negatives?

Adjective Nouns

Seven Negatives
Verb

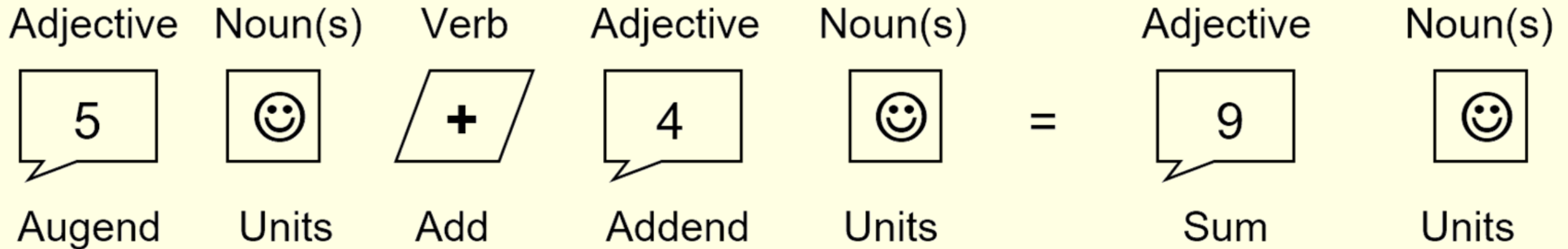
Minus

Adjective Nouns

Four Negatives?

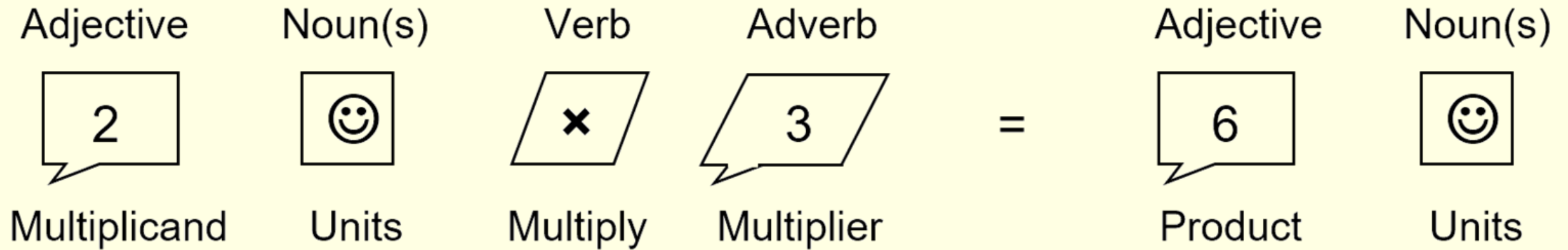
Nouns Make Maths Meaningful!

Addition



Please sign the petition to Teach Indian Maths @
www.j.mp/TeachIndianMaths

Multiplication



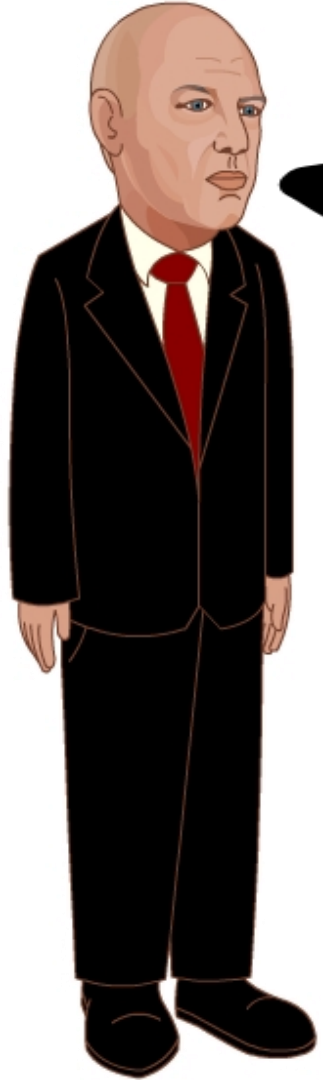
Please sign the petition to Teach Indian Maths @
www.j.mp/TeachIndianMaths

Why the product of two negatives is positive

A demonstration goes like this...

Crabtree, Jonathan J. *Snippet: A new reason negative multiplied by negative is positive* Vinculum, Vol. 52, No. 3, Jul 2015

$1 + (-1) = 0$	Definition of -1 .
$-1 \times [1 + (-1)] = -1 \times 0$	Both sides multiplied by -1 .
$(-1) \times 1 + (-1) \times (-1) = 0$	Distributive law
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$1 + [(-1) + (-1) \times (-1)] = 1 + 0$	Add both sides to 1.
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$0 + (-1) \times (-1) = 1 + 0$	Definition of -1
$(-1) \times (-1) = 1$	Additive identity



Brahmagupta defined zero
as the sum of a positive number
and negative number of equal
magnitude, सम-ऐक्यम् खम् (Brāhma
Sphuta-siddhānta, Chapter 18:30a).

So **zero** was defined in India as:

$$^+n + ^-n$$

2 धनयोर्धनमृणमृणयो-

3 धनार्णयोरन्तरं समैक्यं खम् ।

4 ऋणमैक्यं च धनमृणध-

5 नशून्ययोः शून्ययोः शून्यम् ॥ ३० ॥ (३१)

6 धनयोरैक्यं धनमृणयोरैक्यमृणं भवति । धनार्णयोरन्तरमेकैक्यं भव-
7 ति । समयोर्धनार्णयोरैक्यं खं शून्यं भवति । ऋणशून्ययोरैक्यमृणं धनशू-
8 न्ययोरैक्यं धनं शून्ययोरैक्यं च शून्यं भवति ।

9 अत्रोपपत्त्यर्थं मन्मुद्रिता भास्करबीजटिप्पणी द्रष्टव्या ॥ ३० ॥

10 इदानीं व्यवकलनमाह ।

11 ऊनमधिकाद्विशोध्यं धनं धनादृणमृणादधिकमूनात् ।

12 व्यस्तं तदन्तरं स्यादृणं धनं धनमृणं भवति ॥ ३१ ॥ (३२)

13 शून्यविहीनमृणमृणं धनं धनं भवति शून्यमाकाशम् ।

14 शोध्यं यदा धनमृणादृणं धनाद्वा तदा क्षेप्यम् ॥ ३२ ॥ (३३)

15 अधिकादृणादूनं धनं विशोध्यं शेषं धनं भवति । अधिकादृणादू-
16 नमृणं विशोध्यं शेषमृणं भवति । ऊनादृणादधिकं धनं वानादृणादधिक-
17 मृणं विशोध्यं तदा तदन्तरं व्यस्तं विपरीतं स्यात् । अर्थादधिकं धनं वि-
18 शोध्यं तदा शेषमृणं भवति । अधिकमृणं विशोध्यं तदा शेषं धनं भव-
19 ति । कथं विपरीतं भवतीत्याह । ऋणं धनं भवति धनं चर्णं भवतीति ।
20 चेदृणं शून्यविहीनं शून्येन विहीनं तदा ऋणं धनं च शून्यविहीनं धनं शून्यं
21 च शून्यविहीनमाकाशं शून्यं भवति । यदि ऋणादूनं शोध्यं वा धनादृणं
22 शोध्यं तदा क्षेप्यमर्थात् तदा तयोर्योग एवान्तरं भवतीति ।

23 अत्रोपपत्त्यर्थं मन्मुद्रिता भास्करबीजटिप्पणी विलोक्या ॥ ३१-३२ ॥

24 इदानीं गुणने करणसूत्रम् ।

25 ऋणमृणधनयोर्धातो धनमृणयोर्धनवधो धनं भवति ।

26 शून्यार्णयोः खधनयोः खशून्ययोर्धा वधः शून्यम् ॥ ३३ ॥ (३४)

27 ऋणधनयोर्धात ऋणं भवति । ऋणयोर्वधो धनवधो धनयोर्वधश्च

28 धनं भवति । शून्यार्णयोः खधनयोः शून्यधनयोर्धा खशून्ययोश्च वधः शून्यं

29 भवति ॥ ३३ ॥

30 इदानीं भागहारे करणसूत्रं वृत्तद्वयम् ।

31 धनभक्तं धनमृणहृतमृणं धनं भवति खं खभक्तं खम् ।

32 भक्तमृणेन धनमृणं धनेन हृतमृणमृणं भवति ॥ ३४ ॥ (३५)

33 खोद्धृतमृणं धनं वा तच्छेदं खमृणधनविभक्तं वा ।

34 ऋणधनयोर्वर्गः स्वं खं खस्य पदं कृतिर्यत् तत् ॥ ३५ ॥ (३६)

35 धनं धनभक्तं वा ऋणं ऋणभक्तं फलं धनं भवति । खभक्तं खं

36 फलं खं भवति । ऋणेन धनं भक्तं फलमृणं स्यात् । धनेन ऋणं हृतं फल-
37 मृणं भवति । ऋणं वा धनं खेनोद्धृतं तच्छेदं तस्य शून्यस्य छेदो यस्मि-

38 नृणे वा धने तच्छेदं भवति । एवं खं शून्यमृणधनविभक्तं (शून्यं) वा त-

39 छेदं भवति । फलं शून्यं भवति वा शून्यं तद्वरं स्यादित्यर्थः । ऋणधन-

40 योर्वर्गः स्वं भवति । खस्य वर्गः खं भवति । तदेव वर्गस्य पदं भवति

41 यत्कृतिः स एव वर्गो भवेदिति । भास्करबीजेऽप्येतदेव सर्वम् । अत्र

42 खभक्तं खमर्थात् ० इदं सर्वदा शून्यसमं नेत्येतदर्थं चलनकलनं विलो-

43 क्यम् ॥ ३४-३५ ॥

44 इदानीं सङ्क्रमणविषमकर्माह ।

45 योगोऽन्तरयुतहीनो द्विहृतः सङ्क्रमणमन्तरविभक्तं वा ।

46 वर्गान्तरमन्तरयुतहीनं द्विहृतं विषमकर्म ॥ ३६ ॥ (३७)

47 योगो राश्यायोगोऽन्तरेण राश्यन्तरेण युतो हीनश्च द्विहृतो दलि-

48 तो राशी स्तः । इदं सङ्क्रमणं नाम गणितम् । वा राश्यावर्गान्तरं राश्य-

49 न्तरेण विभक्तं फलमन्तरेण युतं हीनं द्विहृतं च राशी स्तः । इदं विष-

Brahmagupta's 5 Addition Laws

AL1 positive plus positive is positive

AL2 negative plus negative is negative

AL3 positive plus negative is the difference between the positive and negative

AL4 when positive and negative are equal the sum is zero

AL5 positive plus zero is positive

negative plus zero is negative
zero plus zero is zero

Brahmagupta's 4 Multiplication Laws

ML1 The product of a negative and a positive is negative.

ML2 The product of two negatives is positive.

ML3 The product of two positives is positive.

ML4 The product of zero and a negative,
of zero and a positive, or
of two zeros is zero.

Brahmagupta's 5 Subtraction Laws

SL1 A smaller positive subtracted from a larger positive is positive.

SL2 A smaller negative subtracted from a larger negative is negative.

SL3 If a larger negative or positive is to be subtracted from a smaller negative or positive, the sign of their difference is reversed – negative becomes positive and positive negative.

SL4 A negative minus zero is negative,
a positive minus zero is positive,
zero minus zero is zero.

SL5 When a positive is to be subtracted from a negative or a negative from a positive, then it is to be added.

Brahmagupta's 4 Division Laws

DL1 A positive divided by a positive is positive.

DL2 A negative divided by a negative is positive.

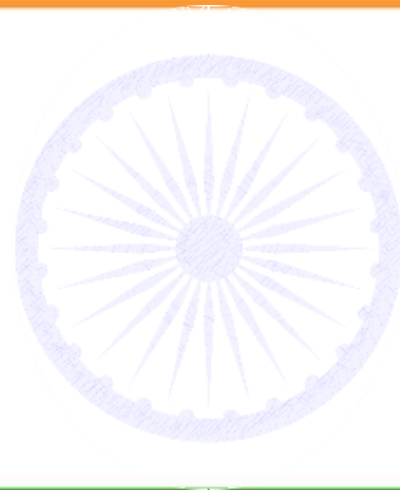
DL3 A positive divided by a negative is negative.

DL4 A negative divided by a positive is negative.

Seeing a Simple Proof for Brahmagupta's ML2

The product of two 'dashed numbers' is positive.

Via Brahmagupta we multiply **positives** and **negatives** by either adding to **zero** multiple times or subtracting from **zero** multiple times.



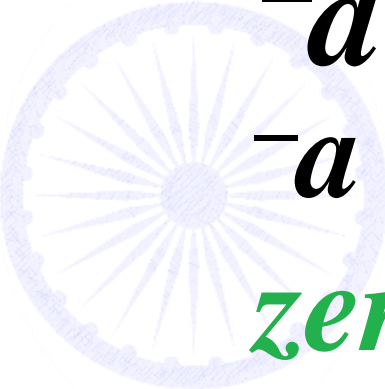
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$-a \times +b =$
 $-a$ *added to*
zero *b times*

$-a \times -b =$
 $-a$ *subtracted from*
zero *b times*



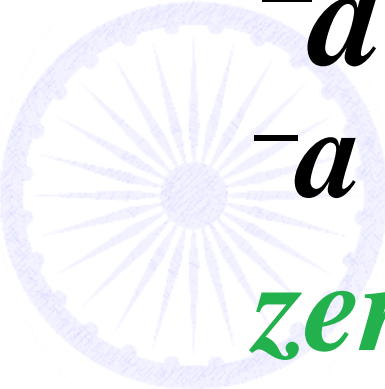
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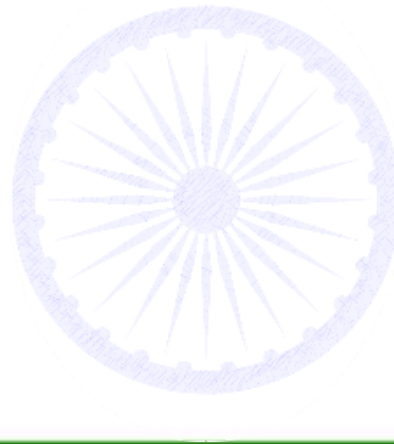
With integral multiplication...

a is the *adjective* describing counts or measures of *noun* quantities
b is the *adverb* describing *verb* counts of additions or subtractions

Seeing a Simple Proof for Brahmagupta's ML2
The product of two 'dashed numbers' is positive.

$-a \times -b = -a$ subtracted from zero b times

$-1 \times -1 = -1$ subtracted from zero 1 time

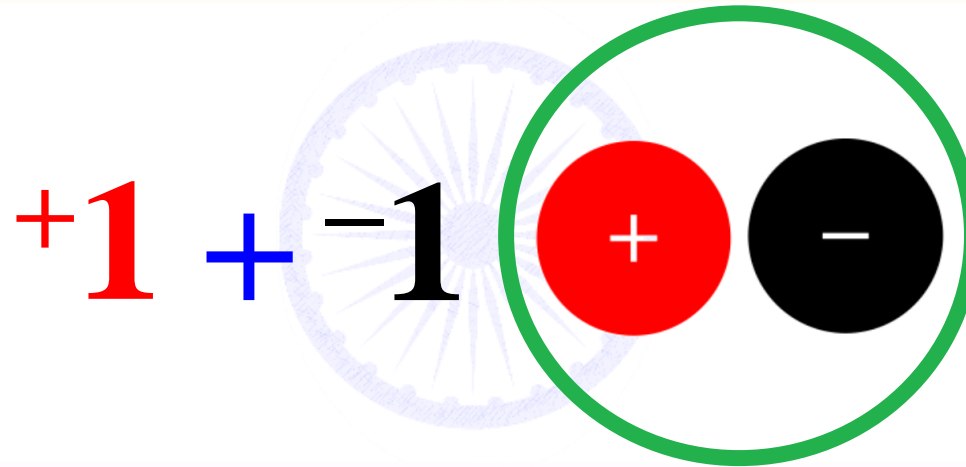


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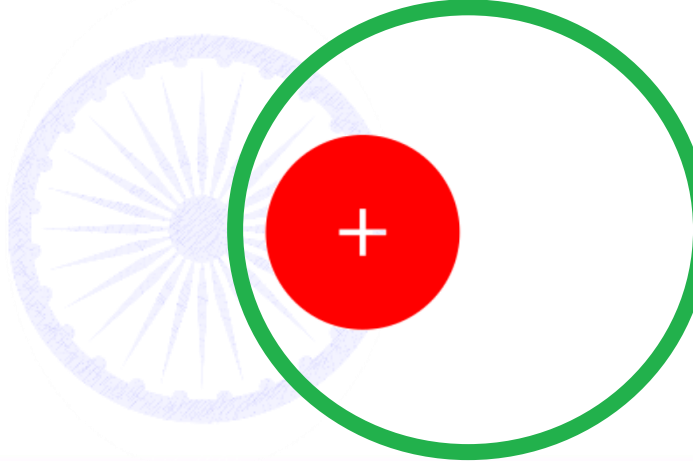
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$(-1) \times 1 + (-1) \times (-1) = 0$	Distributive law
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$1 + [(-1) + (-1) \times (-1)] = 1 + 0$	Add both sides to 1.
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$0 + (-1) \times (-1) = 1 + 0$	Definition of -1
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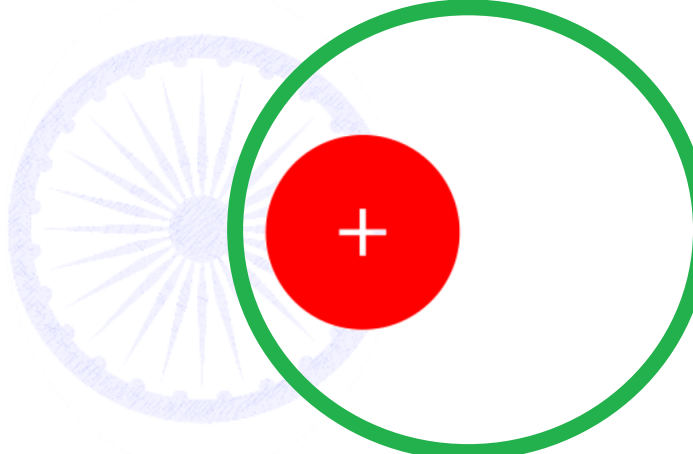
My Sanskrit-Based
Demonstration

Seeing a Simple Proof for Brahmagupta's ML2

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My Sanskrit-Based
Demonstration

My Western
Demonstration

With our corrected understanding of Integer arithmetic, we can now depict it on a Brahmaguptan Plane.



Download the Paper from
www.podometic.in/papers

**n times added to 0
multiplier**

Q. II

Q. I

$-a \times +n = \text{Negative Product}$
i.e. a debt added n times
to zero. $0 + -a + -a + \dots$

$+a \times +n = \text{Positive Product}$
i.e. a fortune added n times
to zero. $0 + +a + +a + \dots$

$-a$
multiplicand
e.g. debt

$+a$
multiplicand
e.g. fortune

$-a$

multiplicand
e.g. debt

$+a$

multiplicand
e.g. fortune

$-a \times -n = \text{Positive Product}$
i.e. a debt subtracted n times
from zero. $0 - -a - -a + \dots$

Q. III

$+a \times -n = \text{Negative Product}$
i.e. a fortune subtracted n times
from zero. $0 - +a - +a + \dots$

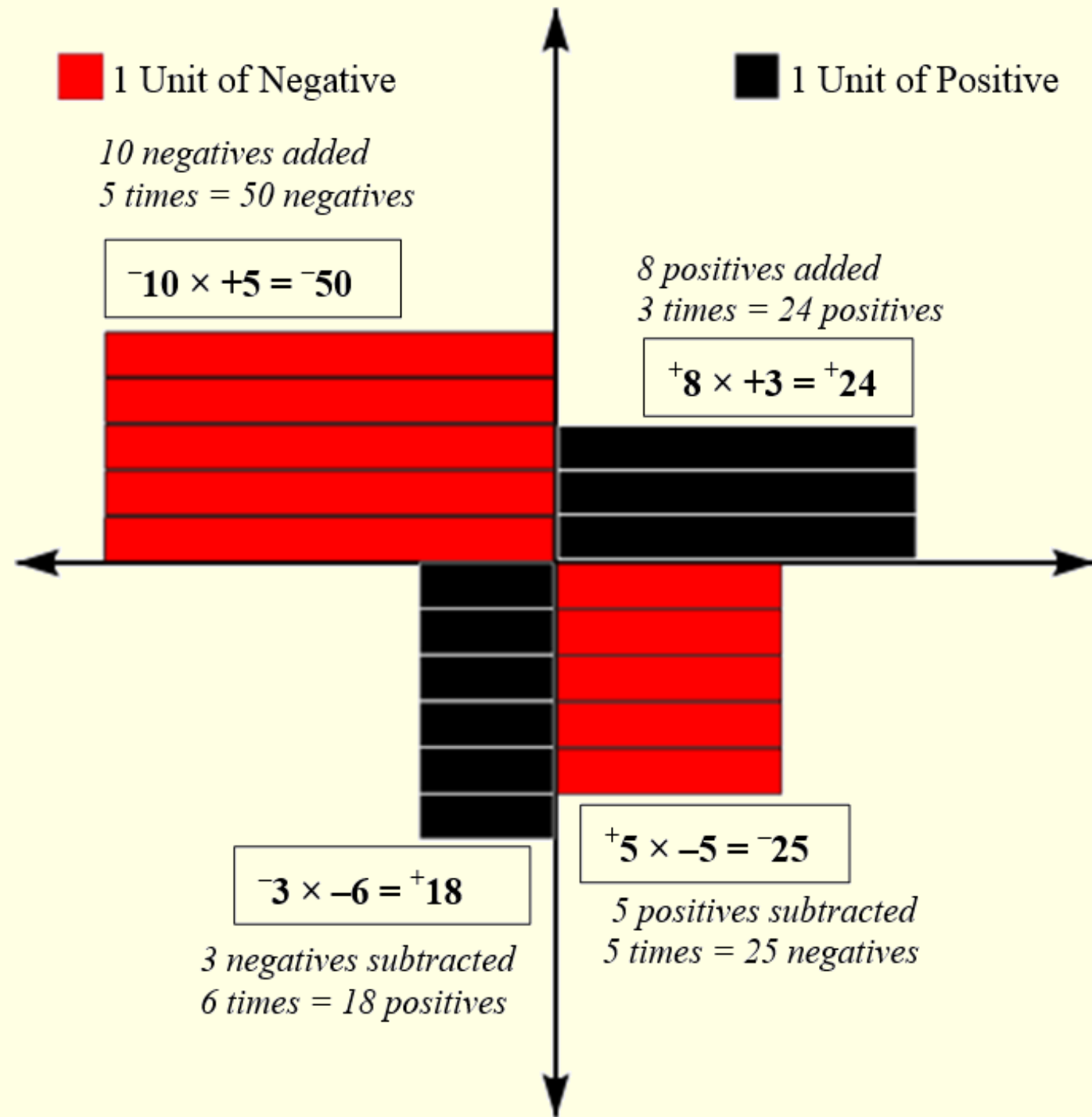
Q. IV

$-n$

multiplier

n times subtracted from 0

The Brahmaguptan Plane with both positive and negative areas.



Jonathan J Crabtree's 'Brahmaguptan Plane' with positive and negative areas.

Addition of Integers to Zero																		
Negatives Added N Times to Zero									Multiplier	Positives Added N Times to Zero								
81	72	63	54	45	36	27	18	9	+9 +8 +7 +6 +5 +4 +3 +2 +1	9	18	27	36	45	54	63	72	81
72	64	56	48	40	32	24	16	8		8	16	24	32	40	48	56	64	72
63	56	49	42	35	28	21	14	7		7	14	21	28	35	42	49	56	63
54	48	42	36	30	24	18	12	6		6	12	18	24	30	36	42	48	54
45	40	36	30	24	18	12	6	5		5	10	15	20	25	30	35	40	45
36	32	28	24	18	12	9	6	4		4	8	12	16	20	24	28	32	36
27	24	21	18	15	12	9	6	3		3	6	9	12	15	18	21	24	27
18	16	14	12	10	8	6	4	2		2	4	6	8	10	12	14	16	18
9	8	7	6	5	4	3	2	1		1	2	3	4	5	6	7	8	9
-9	-8	-7	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6	+7	+8	+9
9	8	7	6	5	4	3	2	1	-1 -2 -3 -4 -5 -6 -7 -8 -9	1	2	3	4	5	6	7	8	9
18	16	14	12	10	8	6	4	2		2	4	6	8	10	12	14	16	18
27	24	21	18	15	12	9	6	3		3	6	9	12	15	18	21	24	27
36	32	28	24	20	16	12	8	4		4	8	12	16	20	24	28	32	36
45	40	36	32	28	24	20	16	5		5	10	15	20	25	30	35	40	45
54	48	42	36	30	24	18	12	6		6	12	18	24	30	36	42	48	54
63	56	49	42	35	28	21	14	7		7	14	21	28	35	42	49	56	63
72	64	56	48	40	32	24	16	8		8	16	24	32	40	48	56	64	72
81	72	63	54	45	36	27	18	9		9	18	27	36	45	54	63	72	81
Negatives Subtracted N Times from Zero									Multiplier	Positives Subtracted N Times from Zero								
Subtraction of Integers from Zero																		

Representations of Negative and Positive Quantities on a 'Brahmaguptan Plane' for India's Primary Classes

Jonathan J Crabtree

www.j.mp/BrahmaguptanPlane

Abstract: *Children's fear of maths is often associated with the introduction of negative numbers. By way of example, asking adult non-mathematicians for the answer to 'negative seven minus negative four' usually results in a wrong answer. However, asking the same question to 12-year-old children in the form What does seven negatives minus four negatives equal? usually results in the right answer. Why is the difference in comprehension so dramatic? In the problematic expression negative seven minus negative four the syntactic structure is adjective adjective verb adjective adjective. With the absence of a noun, the meaning of such maths for most children is lost. Instead, children (and adults) cling to rules memorised without meaning, such as 'two minuses make a plus'. So, what can we do? The answer is simple. We return to 7th Century writings of India, where we discover the astronomer Brahmagupta documented 'adjective-noun' style laws of sign, not for abstract numbers, but for positive quantities, negative quantities and zero. With this insight, we depict simple object-oriented representations of integer arithmetic involving positive and negative quantities. Such a quantitative pedagogy is concrete in nature, yet isomorphic to 'signed numbers.' Therefore, a solid intuitive foundation of integer arithmetic can be laid. Upon this foundation more abstract structures can be built. The integer teaching model that emerges is called the 'Brahmaguptan Plane'.*

PART 2. Why India Must Change its Story

Bad Maths History \Rightarrow Sad Maths Mystery



Jonathan J. Crabtree, Elementary Mathematics Historian, Australia.

9th National conference: Technology & Innovations in Math Education

Biennial Conference of the Mathematics Department of the IIT Bombay.

Jointly organized by IISER, Pune & BATU, Lonere.

Pune India, 27th December 2019

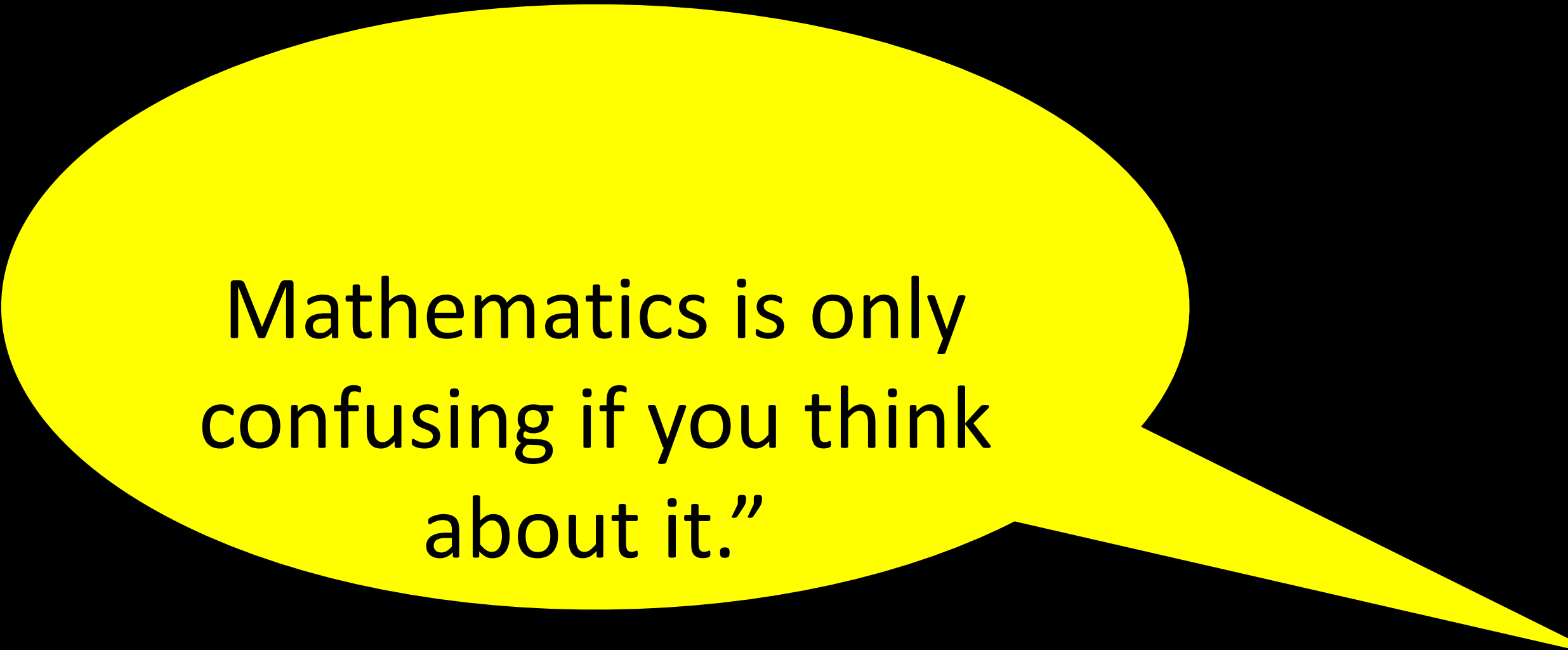
I couldn't make much sense of what
I was being taught. So I felt stupid.

I couldn't make much sense of what
I was being taught. So I felt stupid.



“Don't worry
about it, Jonathan.”

I couldn't make much sense of what
I was being taught. So I felt stupid.



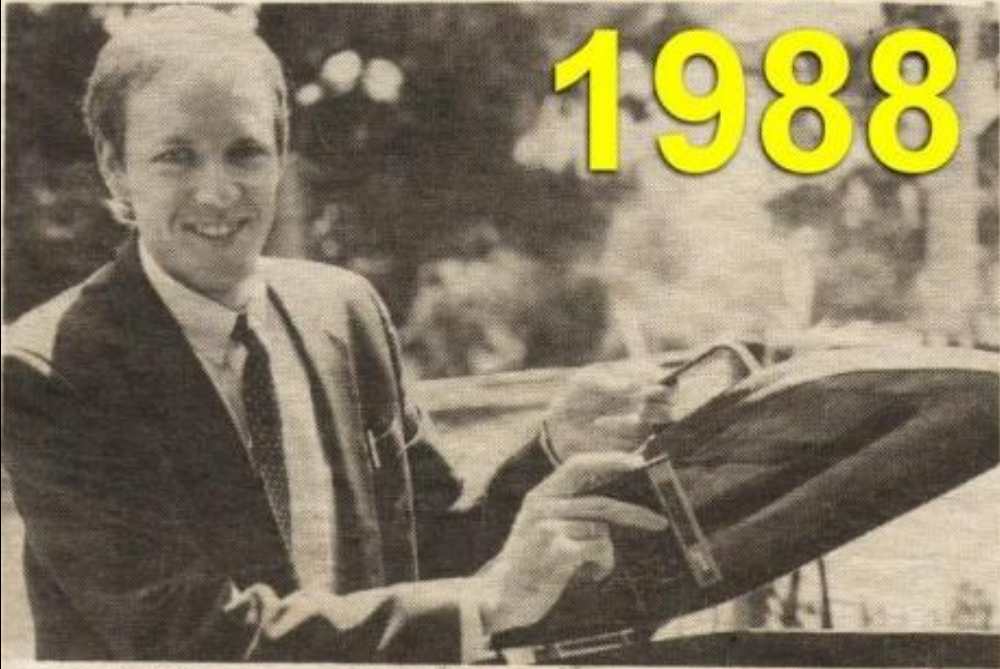
Mathematics is only
confusing if you think
about it."

So, in 1983 I made a decision to change primary
and middle school mathematics explanations.

So, in 1983 I made a decision to change primary and middle school mathematics explanations.

Maths? It's all in the mind, says Jonathan

1988



JONATHAN throws away his calculator and uses brain powers to solve even the hardest of equations.

IF you were asked what day it was on July 24, 1706, what would you say?

It's all in the mind, he says.

After a four second calculation he came up with the correct day.

Jonathan broke the world record for

"I hope to change the way the Western world teaches maths," Jonathon said.

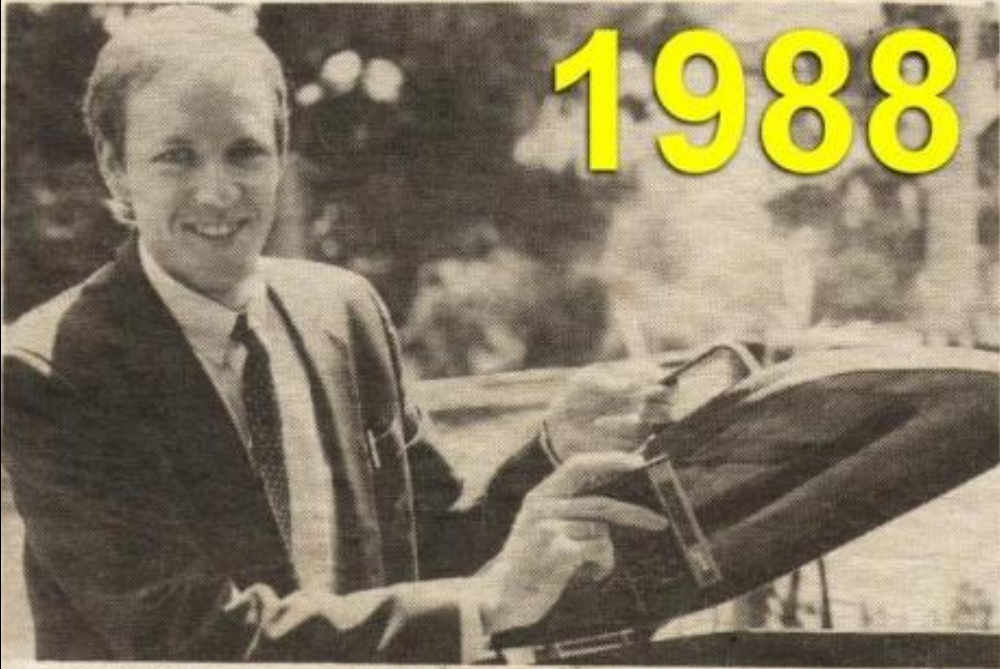
Jonathon will be holding two classes at the Park Orchards Community Centre.

The first is *Memory Unlimited*, which

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At age seven in Class 2, I pulled on a loose thread.

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The multiplication explanation my teacher Miss Collins gave had been wrong for 398 years.

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The multiplication explanation my teacher Miss Collins gave had been wrong for 398 years.

So, that classroom confusion in 1968
led to a peer-reviewed paper.

THE LOST LOGIC OF ELEMENTARY MATHEMATICS AND THE HABERDASHER WHO KIDNAPPED KAIZEN

Jonathan Crabtree **Download the paper @** www.podometric.in/papers

www.jonathancrabtree.com | Mathematics Historian

Euclid's multiplication definition from Elements, (c. 300 BCE), continues to shape mathematics education today. Yet, upon translation into English in 1570 a 'bug' was created that slowly evolved into a 'virus'. Input two numbers into Euclid's step-by-step definition and it outputs an error. Our multiplication definition, thought to be Euclid's, is in fact that of London haberdasher, Henry Billingsley who in effect kidnapped kaizen, the process of continuous improvement. With our centuries-old multiplication definition revealed to be false, further curricular and pedagogical research will be required. In accordance with the Scientific Method, the Elements of western mathematics education must now be rebuilt upon firmer foundations.

At age seven in Class 2, I pulled on a loose thread.

The multiplication explanation my teacher Miss Collins gave had been wrong for 398 years.

So, that classroom confusion in 1968
led to a peer-reviewed paper.

**The more I pulled on loose threads, the
more mathematics education unraveled.**

INDIA WE HAVE A PROBLEM



Many Dislike or Fear Your Maths

INDIA WE HAVE A PROBLEM



Many Dislike or Fear Your Maths
What happened?

The background of the image is a serene coastal scene. In the foreground, there are several large, dark, wet-looking rocks scattered across a sandy beach. The ocean is visible in the middle ground, with gentle waves lapping at the shore. In the far distance, there are low, hazy hills or mountains under a sky that transitions from a warm orange-pink at the horizon to a pale blue at the top. The overall mood is contemplative and peaceful.

**You can't cross the sea
merely by standing and
staring at the water.**

Rabindranath Tagore



**I must travel to find out if there is demand for India's
original and true foundations of mathematics.**

**You can't cross the sea
merely by standing and
staring at the water.**

Rabindranath Tagore





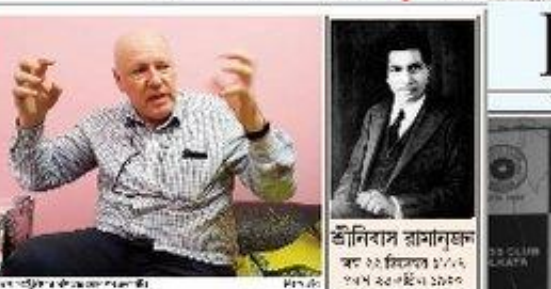


রিলাঞ্চ অফ ইন্ডিয়ান ম্যাথমেটিক্স পর সংগোষ্ঠী



ভাষ্যকার। এসোসিয়েটস অফ ইন্ডিয়ান ম্যাথমেটিক্স প্রকাশন রুই মিথর কে অধ্যাপক রুই এসওয়ার মরদাস কী অধ্যাপক রুই সানিয়ার কী 'রিলাঞ্চ অফ ইন্ডিয়ান ম্যাথমেটিক্স' এর এক সংগোষ্ঠী কা আয়োজন किया यह, जिसमें गणित इतिहासकार जोनाथन जे. क्रॉब्री ने भी भाग लिया था।

শেখবু



রামানুজনের জন্মদিনে সহজ গণিত পদ্ধতি উপহার রাজ্যের প্রতিটি স্কুলে বিনামূল্যে সহজ গণিত শিক্ষার বই দিতে চান অস্ট্রেলিয়ার গণিতজ্ঞ

সহজ গণিত পদ্ধতি উপহার রাজ্যের প্রতিটি স্কুলে বিনামূল্যে সহজ গণিত শিক্ষার বই দিতে চান অস্ট্রেলিয়ার গণিতজ্ঞ

ভীতি কাটাতে শহরে গণিতজ্ঞ

আজকালের প্রতিবেদন



ভীতি কাটাতে শহরে গণিতজ্ঞ

FOTO FACT NEWS



Jonathan J. Crabtree Elementary Mathematics Historian from Melbourne addressing in connection to organise a seminar on "Relaunch of Indian Mathematics" at Rotary Sadan, in association with Indian Red Cross Society and AFX Animation. (Pic. Md. Iqbal Khan)

অস্ট্রেলিয়ান গণিতজ্ঞ জোনাথন জে. ক্রাব্রী তার নিজস্ব গাণিতিক ফর্মুলা চালু হয়েছে বিশ্বের ষাটটি দেশে। কলকাতায় অনুষ্ঠিত হচ্ছে দেশের সবচেয়ে বড় গণিতমেলা। জোনাথন শেখাবেন সহজ গণনা পদ্ধতি

১ থেকে ৫ গুনতে জানলেই জটিল অঙ্ক হবে সহজ, কাটবে ভীতি

সৌগত সরকার

সেই খুজো আজ কল করেছে আপন যদি বলে পঁচ দশটির রকম থাকেন সেটা স্বপ্নীয় বলে দেখে এলাম কলটি ভ্রমি সহজ এবং সৌন্দর্য্য যত্ন পঁচকে খটলে পরে আপনি হবে কোথা।।। সুকুমার রায়ের 'খুজো কল' ছড়ায় চতুর্দশের খুজো এমনই এক আনন্দ বসে বসিয়েছেন। খুজো কল ছিল খুজোর মজিরের থেকেও জটিল। কিন্তু সত্যি যদি এমন কোনও কল থাকত, জটিলতম সমস্যা এক তুড়িতেই গলে জল। বৈজ্ঞানিক সমীক্ষায় দেখা গিয়েছে— পৃথিবীর নব্বই শতাংশ পদার্থের অঙ্ককে ভর পায়, সে ভয়ের কারণ শুধুমাত্র ভীতি এবং অভিজ্ঞতা। প্রায় নব্বই দিন হাজার বছর হয়ে মানব জীবনে গণিতকে কয়েক দশকের জন্য গণিতবিজ্ঞান মাথা ঘামিয়ে আসছেন। সমস্তা যত এগিয়েছে গণিতের প্রয়োজনও তত বেড়েছে। সেই সঙ্গে



"Brahmagupta's 18 laws of mathematics are completely missing from India's present mathematics curriculum."



Jonathan J. Crabtree

কারণ প্রাচীন মনে শব্দের অর্থপ্রিয় ভ্রমি ব্যাখ্যা করবেন জোনাথন। এতকালের গুণের ব্যক্তিগত উল্লেখ এই মহান এবং জ্ঞান। জোনাথন ক্রাব্রী, তার এই বইটি ভারতীয় গণনাশাস্ত্র এবং গণিতশাস্ত্র। প্রাচীন ভারতীয় পুঁথি। পঁচ বছর এই পুঁথি মাথা বিধিবিধিমাণের কাঁচেরে পেতে। কোর অধ্যাপক এই তুলি গণিতের পুঁথির ব্যবহার, গণনাগণ ও গাণিতিক পদ্ধতি পুঁথির মূল বিজ্ঞ। প্রাচীন ভারতীয় গণিতজ্ঞ শুধু সেই গণিতশাস্ত্রেই জারিত হয়েছে। নতুন গণনা পদ্ধতি বিভিন্নদেয় পুঁথিগত লোক ভাবে চালায় আছে। সমস্তা পঁচ এই গুণে সমস্তা (বিশ্ববাস মেডেল) যা নোবেলের পুরস্কারে। কিন্তু জোনাথন এবং এতকালের গণিতের বই পুঁথিগত স্টেডি হয়ে মানব মন হজ গণনা পদ্ধতি অঙ্ককে সহজ করে দেবে।

Arithmetic?

Basic Operations (+, −, ×, ÷) on

Arithmetic?

Basic Operations (+, −, ×, ÷) on

... −3 −2 −1 0 +1 +2 +3 ...

**Obviously, India's ancient
integer logic got to us today.**

Obviously, India's ancient
integer logic got to us today.

But how?

What FACTS do we know?

What FACTS do we know?

India's definition of zero as a number and Integer arithmetic was embraced by the Arabic world HAN

What FACTS do we know?

India's definition of zero as a number and Integer arithmetic was embraced by the Arabic world HAN

Al-Khwārizmī wrote a book on Hindu Integer arithmetic which featured Brahmagupta's ancient laws of sign for negatives and positives

What FACTS do we know?

India's definition of zero as a number and Integer arithmetic was embraced by the Arabic world HAN

Al-Khwārizmī wrote a book on Hindu Integer arithmetic which featured Brahmagupta's ancient laws of sign for negatives and positives

Based on what he learned from the Indians, al-Khwārizmī then wrote a book on algebra

Al-Khwārizmī's **algebra** text c. 820 CE

- لكتاب المختصر في حساب الجبر والمقابلة
- Al-Kitāb al-mukhtasar fī hisāb **al-jabr** wa'l-muqābala
- The Compendious Book on Calculation by Completion and Balancing.

What FACTS do we know?

Arabic writers understood **negative** terms. E.g.

In mathematical language, the verb [jabr] means...
... to transpose **negative** quantities to the opposite side by changing their signs. The **negative** quantity thus removed...

Rosen 1831: p. 178

What FACTS do we know?

Arabic writers understood **negative** terms. E.g.

The usual meaning of jabr in mathematical treatises is: adding equal terms to both sides of an equation in order to eliminate **negative** terms.

Van der Waerden 1985: p. 4

What FACTS do we know?

Arabic writers understood **negative** terms. E.g.

Al-jabr means “restoration” or “completion”, that is, removing **negative** terms, by transposing them to the other side of the equation to make them positive

Devlin 2012: p. 53

What FACTS do we know?

From the Arabic world, India's mathematical foundations made their way to North Africa where Leonardo Pisano (AKA Fibonacci) mastered them

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From the Arabic world, India's mathematical foundations made their way to North Africa where Leonardo Pisano (AKA Fibonacci) mastered them

Leonardo Pisano then documented India's mathematical foundations involving Brahmagupta's definition of zero as a number

What FACTS do we know?

From the Arabic world, India's mathematical foundations made their way to North Africa where Leonardo Pisano (AKA Fibonacci) mastered them. Leonardo Pisano then documented India's mathematical foundations involving Brahmagupta's definition of zero as a number.

Thus, Europe came to understand Indian arithmetic.

The Transmission of India's Integer Arithmetic

India
7th century



Shadings for illustrative purposes only

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Reserved www.podometic.in

The Transmission of India's Integer Arithmetic

India
7th century



Arabic World
9th century



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The Transmission of India's Integer Arithmetic

India
7th century



Arabic World
9th century



Europe
13th century



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What FACTS do we teach?

Zero is defined as $n - n$

What FACTS do we teach?

Zero is defined as $n - n$

Negative numbers are less than zero

What FACTS do we teach?

Zero is defined as $n - n$

Negative numbers are less than zero

Negative seven is less than negative four $-7 < -4$

What FACTS do we know?

Zero is defined as $n - n$

Negative numbers are less than zero

Negative seven is less than negative four $-7 < -4$

Every basic arithmetical operation (+, −, ×, ÷) on the Integers is understood and has been for centuries

What FACTS do we teach?

Euclid in his book *Elements* defined multiplication as repeated addition

What FACTS do we teach?

Euclid in his book *Elements* defined multiplication as repeated addition

ab is thus defined as ***a*** added to itself ***b*** times

What FACTS do we teach?

Euclid in his book *Elements* defined multiplication as repeated addition

ab is thus defined as a added to itself b times

a^b is thus defined as a into itself b times

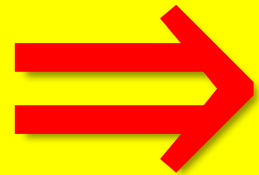
**What we know about
arithmetic isn't true!**

**What we know about
arithmetic isn't true!**

**Every previous
FACT is FALSE**

Bad Maths History

Bad Maths History



Sad Maths Mystery

Indian students rank 2nd last in global test

TNN | Jan 15, 2012, 02.24 AM IST



School students celebrate after checking their CBSE results. A global survey has found that the average 15-yea... [Read More](#)

MUMBAI: Across the world, India is seen as an education powerhouse — based largely on the reputation of a few islands of academic excellence such as the IITs. But scratch the glossy surface of our education system and the picture turns seriously bleak.

Fifteen-year-old Indians who were put, for the first time, on a global stage stood second to last, only beating Kyrgyzstan when tested on their reading, math and science abilities.

India ranked second last among the 73 countries that participated in the Programme for International Student Assessment (PISA), conducted annually to evaluate education systems worldwide by the OECD (Organisation for Economic Co-operation and Development) Secretariat. The survey is based on two-hour tests that half a million students are put

through.

Tamil Nadu and Himachal, showpieces of India's education and development, fared miserably at the Programme for International Student Asssment, conducted by the Organisation for Economic Co-operation and Development Secretariat.

In math, considered India's strong point, they finished second and third to last, beating only Kyrgyzstan

Bad Maths History

Sad Maths Mystery

Everyone is entitled to their own opinion, yet not to their own facts.

Extraordinary Claims

- India's definition of **ZERO** never made it to either the ancient Arabic world or Europe.

Extraordinary Claims

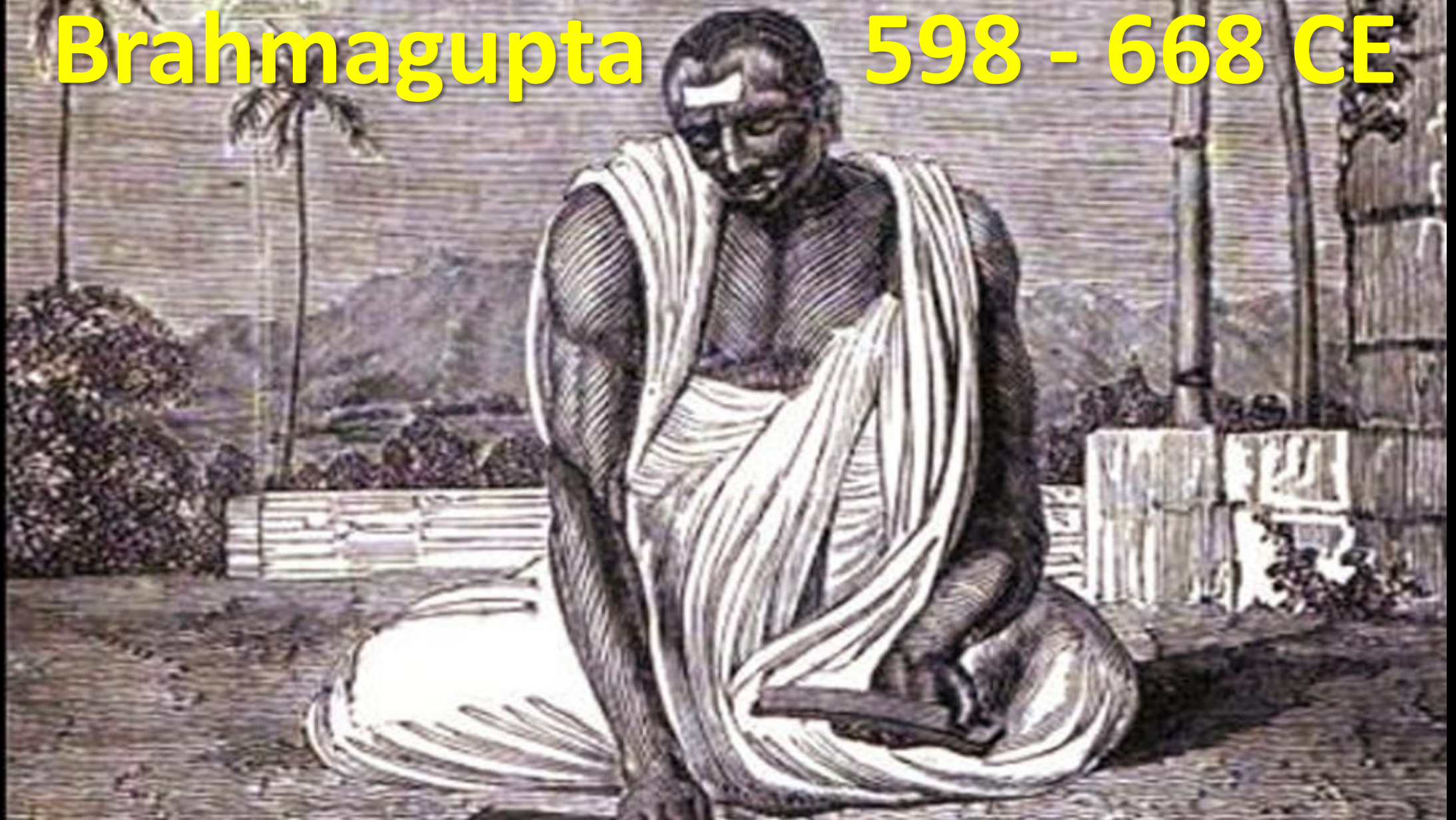
- India's definition of **ZERO** never made it to either the ancient Arabic world or Europe.
- In the Arabic world, India's **ZERO** only came to exist as a placeholder, not as the power tool to solve simple problems like $+3$ minus $+4$, or -2 minus -4 , or -4 minus $+2$

Extraordinary Claims

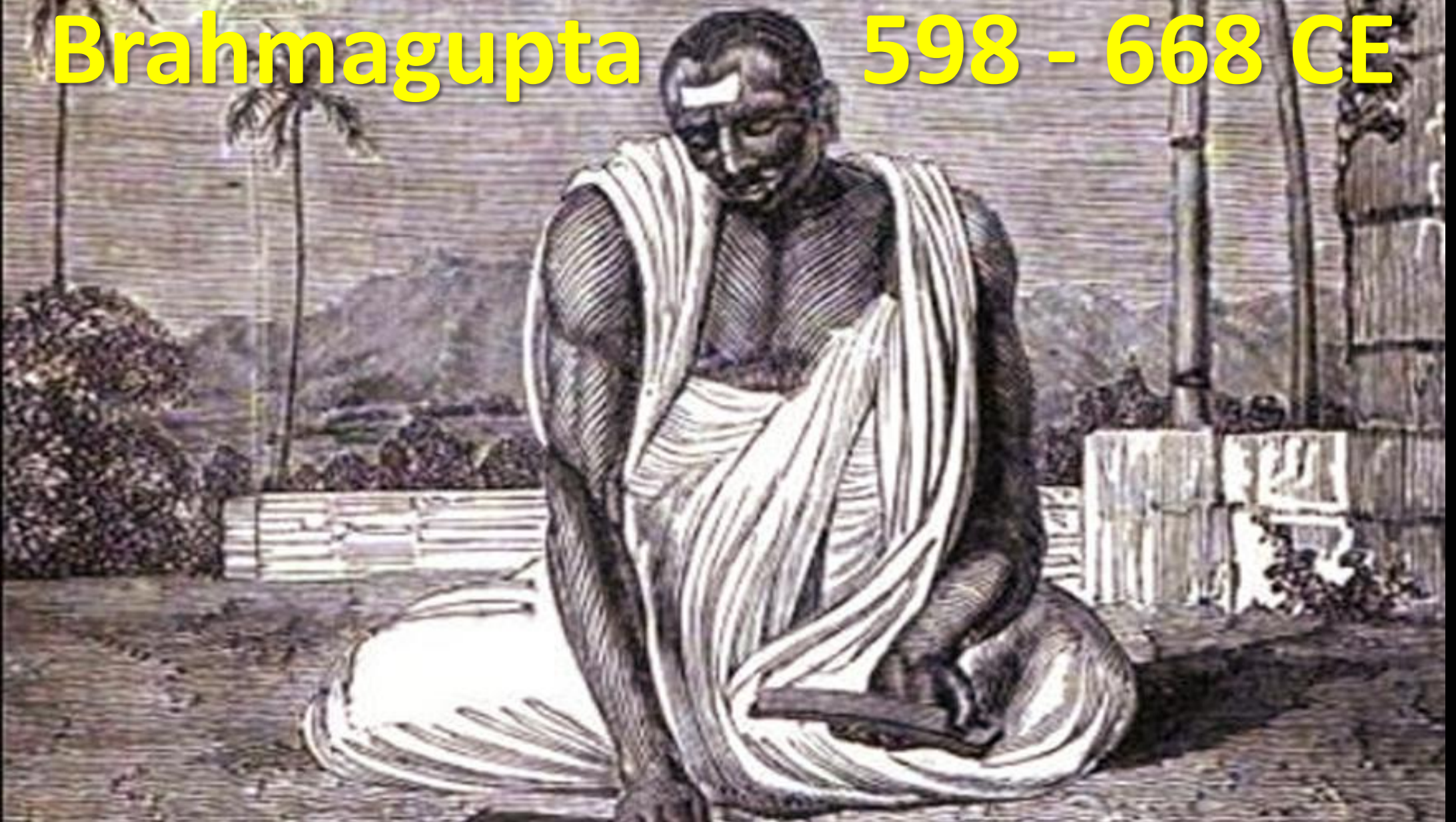
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Extraordinary Evidence...

Brahmagupta 598 - 668 CE

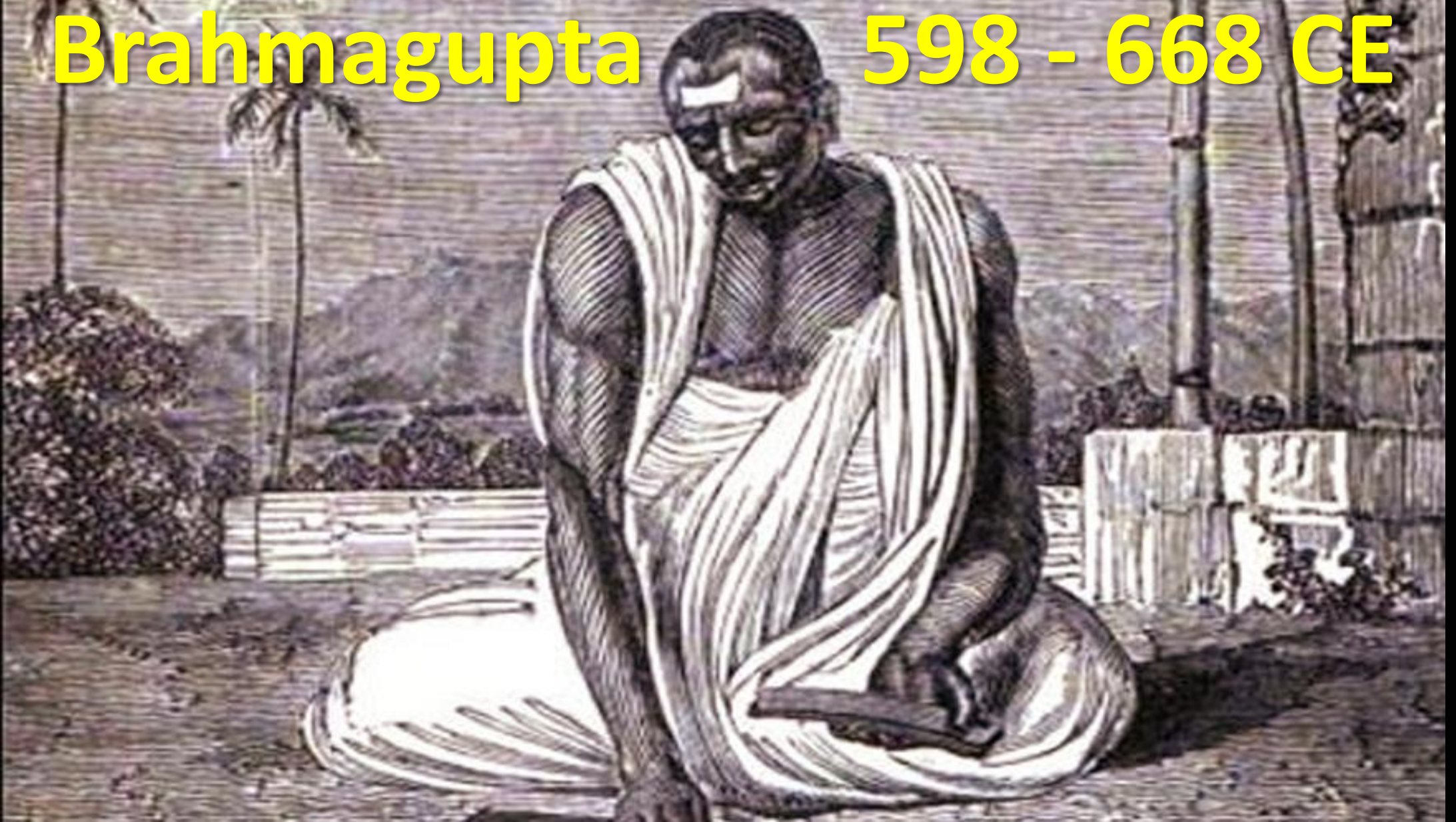


Brahmagupta 598 - 668 CE



Astronomer and Mathematician

Brahmagupta 598 - 668 CE



Brāhmasphuṭasiddhānta 628 CE

For this talk,
Brahmagupta's
Laws of Positives
Negatives and
Zero have been
freshly analysed.

Images courtesy of
the British Library.

REPRINT FROM THE PANDIT.

ब्राह्मस्फुटसिद्धान्तो

ध्यानग्रहोपदेशाध्यायश्च ।

गणकचक्रचूडामणिश्रीब्रह्मगुप्तविरचितः ।

महामहोपाध्यायसुधाकरद्विवेदिकृतनूतन-
तिलकसमेतः ।

BRĀHMASPHUṬASIDDHĀNTA

AND

DHYĀNAGRAHOPADEŚĀDHYĀYA,

BY BRAHMAGUPTA,

EDITED WITH HIS OWN COMMENTARY

BY

MAHĀMAHOPĀDHYĀYA SUDHĀKARA DVIVEDIN,

Professor, Queen's College, Benares.



BENARES:

PRINTED AT THE MEDICAL HALL PRESS.

1902.

Brahmagupta's 5 Addition Laws AL (*saṅkalana*)

Brahmagupta's 5 Addition Laws

AL1 positive plus positive is positive

AL2 negative plus negative is negative

AL3 positive plus negative is the difference between the positive and negative

AL4 when positive and negative are equal the sum is zero

AL5 positive plus zero is positive
negative plus zero is negative
zero plus zero is zero

Brahmagupta's 5 Addition Laws

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Brahmagupta's 5 Addition Laws

AL1 positive plus positive is positive Understood in Arabic world

AL2 negative plus negative is negative

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AL4 when positive and negative are equal the sum is zero

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negative plus zero is negative
zero plus zero is zero



Al-Khwārizmī (c. 780-850)

I had seen that the Indians had set up **9 symbols** in their universal system of numbering...



Al-Khwārizmī did not
mention zero

Al-Khwārizmī (c. 780-850)

So they made **9 symbols**, which are
these: 9 8 7 6 5 4 3 2 1.

And ... **every number is put together
above one.**



Al-Khwārizmī did not mention zero and did not consider one a number.

Al-Khwārizmī (c. 780-850)

... **one** is the root of all number and **is outside number**.

It is the root of number because every number is found by it.

But it [one] is outside number because it is found by itself, I mean, without any other number.

Al-Uqlidisi (c. 920-980)



**Al-Uqlidisi considered
zero a placeholder,
not a number.**



Al-Uqlidisi considered zero an empty placeholder, not a number.

Al-Uqlidisi (c. 920-980)

One question is: **Why are the Hindi letters nine**, no more, no less? We say: Because the beginning of numbers from which they start is one and the last unit we pronounce is nine. Thus when we say units we mean something between one and nine; after that units are over, and ten comes out like one and takes its form. We add up ten to ten until we reach 90 which conforms with nine. Tens are now over and we say one hundred, coming back to one, and going up to 9. Thus we see that all places start with one and end with nine. That is why they are made nine. **So much for the nine letters.** If it is said: Why is zero multiplied by zero equal to zero and zero multiplied by any letter zero? We say that by multiplying zero by **zero the aim is only to occupy the place**; the same applies for multiplying the letter by zero. **We multiply the letter by zero only once, the first time, by the first letter in the upper, to occupy the place, and tell that there is a place and that it is empty.**



200 years after Brahmagupta,
al-Khwārizmī did not accept 1 as a
number. Zero as a number? Never!



200 years after Brahmagupta,
al-Khwārizmī did not accept 1 as a
number. Zero as a number? Never!



300 years after Brahmagupta,
al-Uqlīdisī accepted India's **ZERO** as a
placeholder, yet not a number. Why?



Al-Uqlīdisī means 'the Euclidist'. He was known for his skill in studying the Greek geometry of Euclid and translating it into Arabic.



Al-Uqlīdisī means 'the Euclidist'. He was known for his skill in studying the Greek geometry of Euclid and translating it into Arabic.

Around 300 BCE, Euclid defined 'number' as *a multitude of units*. So Euclid's definition of number came before 0 and 1 were numbers.

India defined zero as the sum of opposing negative and positive numbers or quantities with the same multitude or magnitude.

India defined zero as the sum of opposing negative and positive numbers / quantities with the same multitude or magnitude.

If Arabic and European writers in medieval times *really* understood India's zero, where are all the negative numbers in their writings?

“I have read a few dozen medieval Arabic books on arithmetic and algebra, and there is no hint of negative numbers in any of them. Zero, too, was not regarded to be a number, but was merely the place holder for an empty place in the representation of a number in Arabic (Indian) notation.”

**By email courtesy of Dr. Jeffrey Oaks, Professor of Mathematics
Medieval Arabic algebra and the mathematics of Greece and medieval Europe UINDY**

“I have read a few dozen medieval Arabic books on arithmetic and algebra, and there is no hint of negative numbers in any of them. Zero, too, was not regarded to be a number, but was merely the place holder for an empty place in the representation of a number in Arabic (Indian) notation.”

“All numbers in Arabic arithmetic were positive. No Arabic author to my knowledge ever even contemplated the existence of negative numbers.”

**By email courtesy of Dr. Jeffrey Oaks, Professor of Mathematics
Medieval Arabic algebra and the mathematics of Greece and medieval Europe UINDY**

The maths MYTHS we know

India's definition of zero as a number that was a sum of equal, yet opposite negative and positive quantities was **[NOT]** embraced by the Arabic world

The maths MYTHS we know

India's definition of zero as a number that was a sum of equal, yet opposite negative and positive quantities was [NOT] embraced by the Arabic world

Al-Khwārizmī wrote a book on Hindu Integer arithmetic which **[DID NOT]** feature Brahmagupta's ancient laws of sign for negatives and positives

The maths MYTHS we know

Based on what he learned and documented in his book *Algoritmi de numero Indorum* (al-Khwārizmī on the Hindu Art of Reckoning) he then wrote a book on algebra **[NO]**

Al-Khwārizmī wrote his book on algebra BEFORE he understood Indian mathematics.

The maths MYTHS we know

Al-Khwārizmī did **NOT** remove negative terms from his equations.

The maths MYTHS we know

Al-Khwārizmī did **NOT** remove negative terms from his equations.

Al-Khwārizmī simply eliminated any positive term that was being subtracted in an equation.

The maths MYTHS we know

Al-Khwārizmī did **NOT** remove negative terms from his equations.

Al-Khwārizmī simply eliminated any positive term that was being subtracted in an equation.

For example, $ax^2 = bx - c$ became $ax^2 + c = bx$.

The maths MYTHS we know

Comparing al-Khwārizmī's approach to Brahmagupta's,
Rashed, Roshdi. (2009) *The Beginnings of Algebra*. Saqi, London.

The maths MYTHS we know

Comparing al-Khwārizmī's approach to Brahmagupta's

Rashed, Roshdi. (2009) *The Beginnings of Algebra*. Saqi, London.

“Once again al-Khwārizmī differs from Brahmagupta, this time in not employing any abbreviation.

The maths MYTHS we know

Comparing al-Khwārizmī's approach to Brahmagupta's
Rashed, Roshdi. (2009) *The Beginnings of Algebra*. Saqi, London.

Al-Khwārizmī... avoids using “negative” numbers or simply a [larger] number subtracted from a smaller one, or from zero, whereas Brahmagupta, like other Indian mathematicians before him, does not hesitate to make use of such [negative] numbers.”

The maths MYTHS we know

“It is difficult to imagine that al-Khwārizmī, if he had read this chapter [i.e. chapter 18 of Brahmagupta’s Brāhma Sphutasiddhānta] would not have been able to profit by it, even if only to shorten the presentation of his work.”

The maths MYTHS we know

“It is difficult to imagine that al-Khwārizmī, if he had read this chapter [i.e. chapter 18 of Brahmagupta’s Brāhma Sphutasiddhānta] would not have been able to profit by it, even if only to shorten the presentation of his work.”

“The style of the mathematical reasoning that is at work in al-Khwārizmī’s algebra has nothing to do with what we encounter in the work of his (Indian) predecessors.”

**628 CE
Brahmagupta
had everything
we need today!**



628 CE
Brahmagupta
had everything
we need today!



830 CE
Al-Khwārizmī
did not have 1
as a number.

628 CE
Brahmagupta
had everything
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830 CE
Al-Khwārizmī
did not have 1
as a number.



950 CE
Al-Uqlīdisī
only had 0 as
a placeholder

628 CE
Brahmagupta
had everything
we need today!

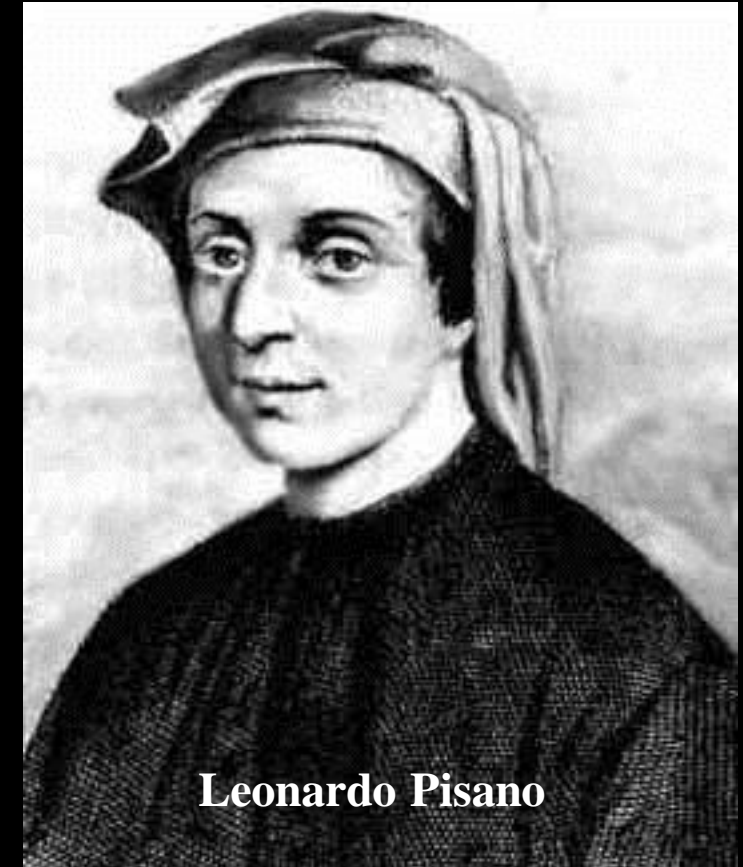


830CE
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did not have 1
as a number.



950 CE
Al-Uqlīdisī
only had 0 as
a placeholder

Leonardo Pisano
1170 – 1250



Leonardo Pisano

628 CE
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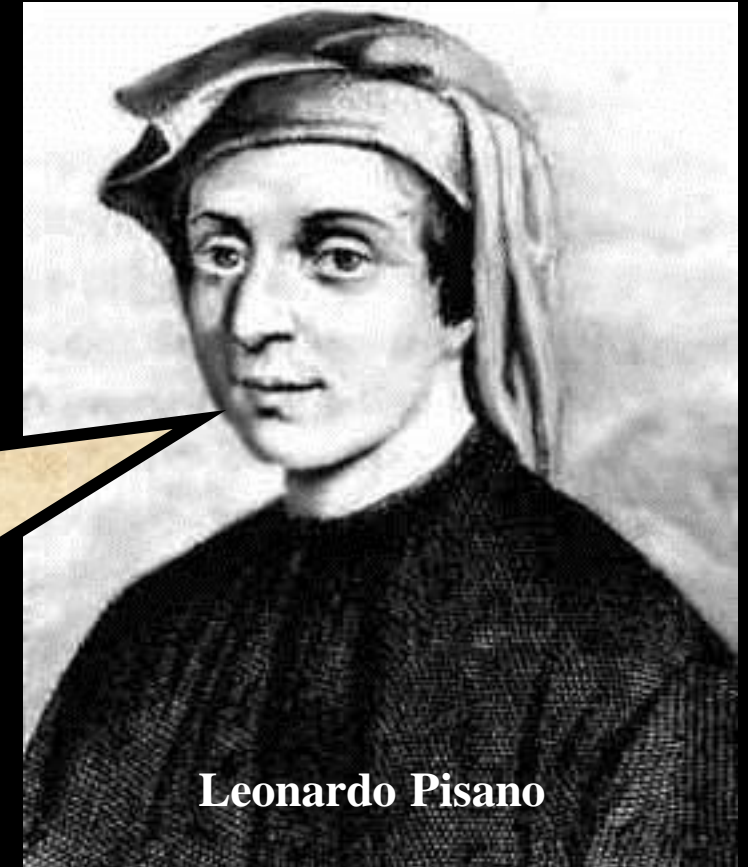


830CE
Al-Khwārizmī
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as a number.



950 CE
Al-Uqlīdisī
only had 0 as
a placeholder

**I am Leonardo Pisano. I am the
man most responsible for
introducing India's arithmetic
into Europe in the 13th Century
via my book Liber Abaci.**



Leonardo Pisano

628 CE

**Brahmagupta
had everything
we need today!**

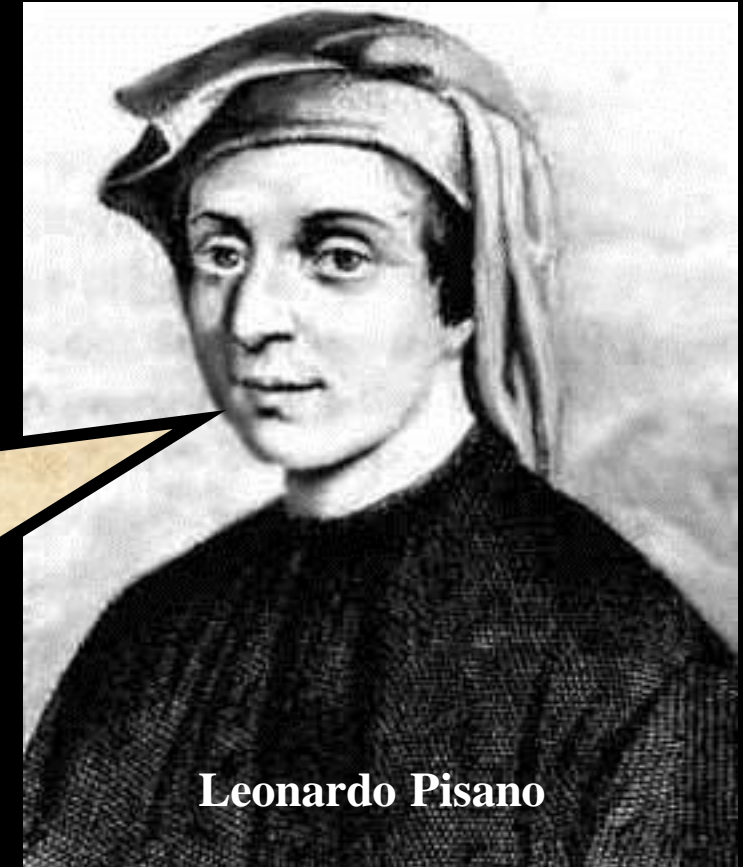


830CE
Al-Khwārizmī
did not have 1
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950 CE
Al-Uqlīdisī
only had 0 as
a placeholder

As I got my Indian info via
Arabic traders, I did **NOT** get to
learn about India's definition of
zero as a number or the rules of
positive and negatives.



Leonardo Pisano

628 CE
Brahmagupta
had everything
we need today!

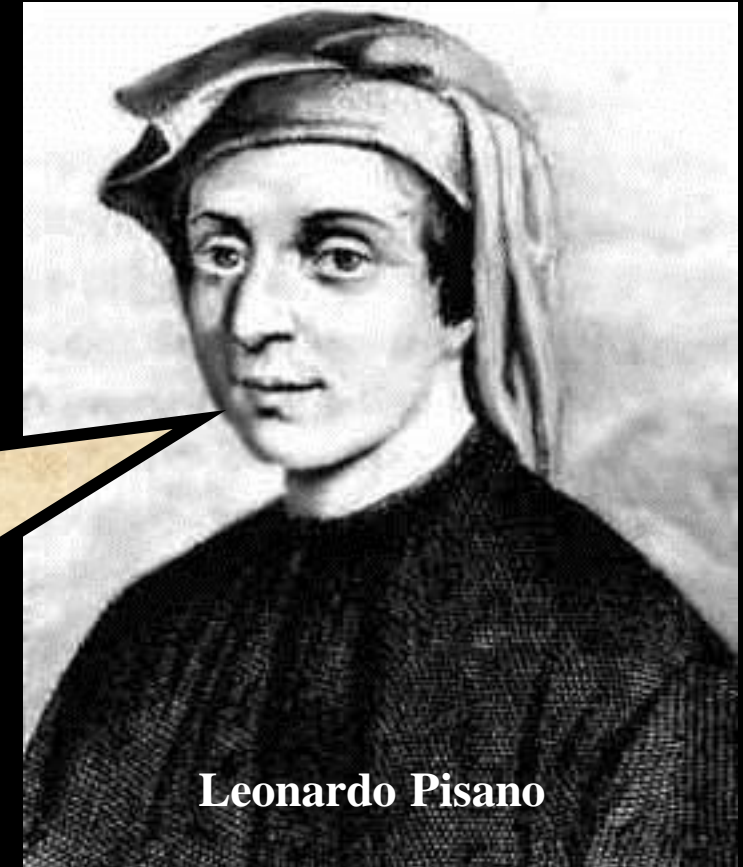


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As I got my Indian info via
Arabic traders, I did **NOT** get to
learn about India's definition of
zero as a number or the rules of
positive and negatives. **Whoops!**



Leonardo Pisano



Transmission of zero
as a placeholder, yet
not as defined by
Brahmagupta 628 CE.





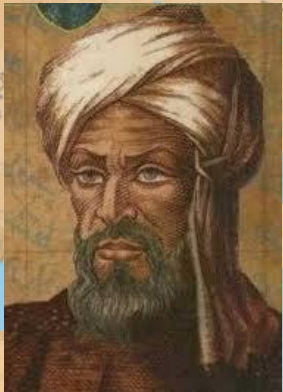
Transmission of zero as a placeholder, yet not as defined by Brahmagupta 628 CE.





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Transmission of zero as a placeholder, yet not as defined by Brahmagupta 628 CE.





Transmission of zero as a placeholder, yet not as defined by Brahmagupta 628 CE.

Those unaware zero was defined as the sum of equal pos. & neg.

- Al-Khwārizmī, Iraq 9th C.
- Traders, Nth Africa 12th C.
- Leonardo Pisano, Italy 13th C.
- Robert Recorde, England 16th C.

I invented the sign = and introduced the pre-existing sign + to England.



Transmission of zero as a placeholder, yet not as defined by Brahmagupta 628 CE.

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Yet I never knew about your zero definition Mr Brahmagupta or laws of positives & negatives

Recordes did know subtraction subtracted is addition yet that's a different idea.



Transmission of zero as a placeholder, yet not as defined by Brahmagupta 628 CE.

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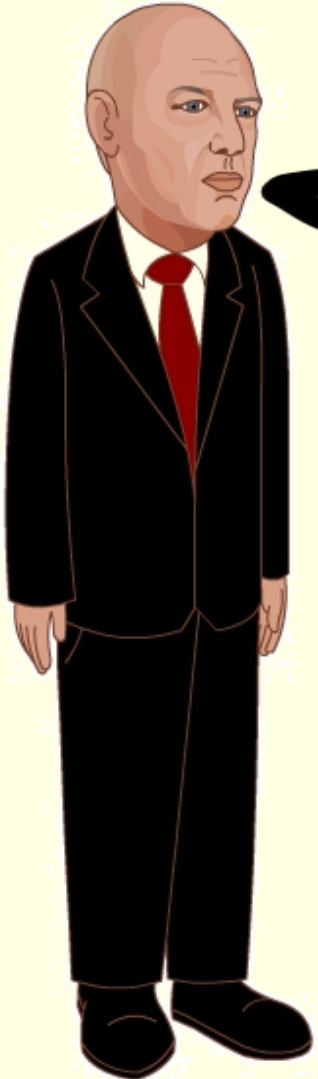
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**In 1478 the first book printed
on maths (Treviso Arithmetic)
said numbers start at 2.**

**So much for 0 and 1 which is
all your computer needs!**

www.podometic.in



**In 16th Century England
people used Roman
Numerals and there was
no Roman Numeral for 0.**

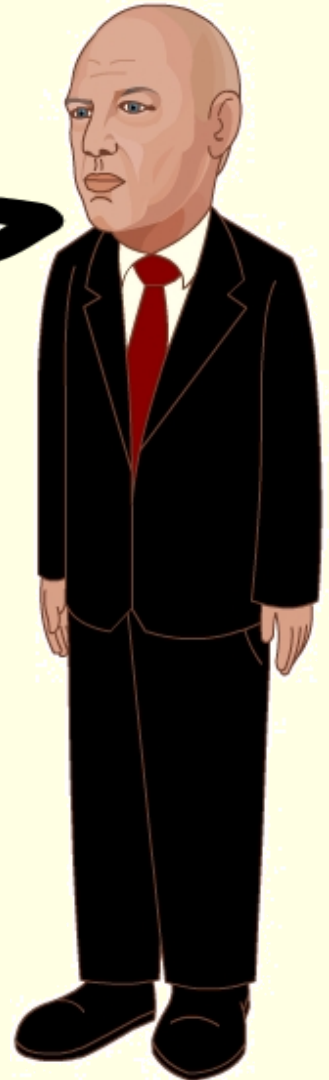


**The maths at the time
was based on Ancient
Greek maths which did
not have zero, one or
negative numbers.**

The false idea negative quantities are less than zero (rather than opposite in nature to positive quantities) emerged in Michael Stiefel's *Arithmetica Integra* of 1544 in a section titled De signis additorum & subtractorum & de numeris absurdis.



**Michael Stiefel said negative
numbers were below zero
which is below nothing *infra 0,*
id est infra nihil and negative
numbers were absurd *numeri*
*absurdi.***



To make sense of numbers that count or measure negative quantities, (i.e. negative numbers) all we need to do is drop the nonsense notion that negative quantities are 'less than zero'.



**Then negative numbers
simply count or measure
opposite quantities or
forces, which are always
greater than zero!**



**Perhaps you might recall
Newton's Third Law which
states for every action there
is an equal and opposite
reaction. Bingo!**



**Newton's Laws of Motion
are consistent with
Brahmagupta's laws of
quantitative mathematics,
which are also consistent
with quantum physics.**



**Think about it... 3 negative
electrons and 3 positive
positrons cancel each
other out to sum to zero.**





As maths books got published in the English language, (without 0 or 1 in algorithmic definitions or as numbers) they were exported to England's settlements and colonies (e.g. New England became America).

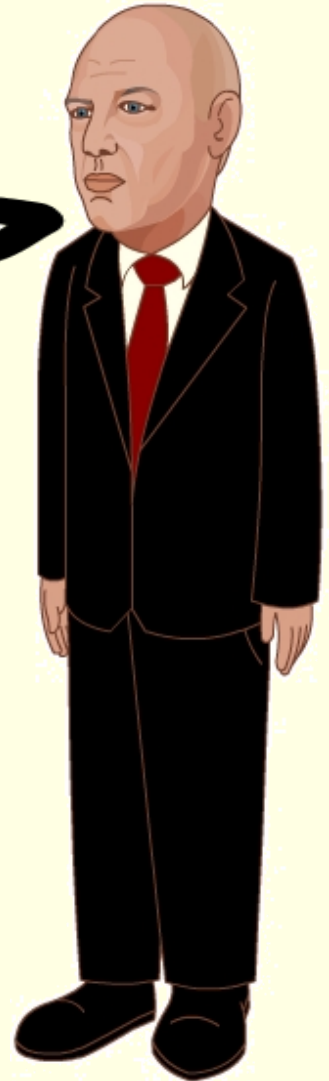


So as the English language spread, so too did **major misunderstandings** of India's mathematical foundations!

In 628 Brahmagupta gave
solutions to equations we'd
write today as $x^2 - 92y^2 = 1^*$
and $ax^2 + bx + c = 0$.



In 628 Brahmagupta gave
solutions to equations we'd
write today as $x^2 - 92y^2 = 1^*$
($x = 1151$ and $y = 120$)



However, the first person to say 1 was a number in the West was Simon Stevin in 1585, almost 1000 years after Brahmagupta!



However, the first person to say 1 was a number in the West was Simon Stevin in 1585, almost 1000 years after Brahmagupta! **So, where is zero today?**



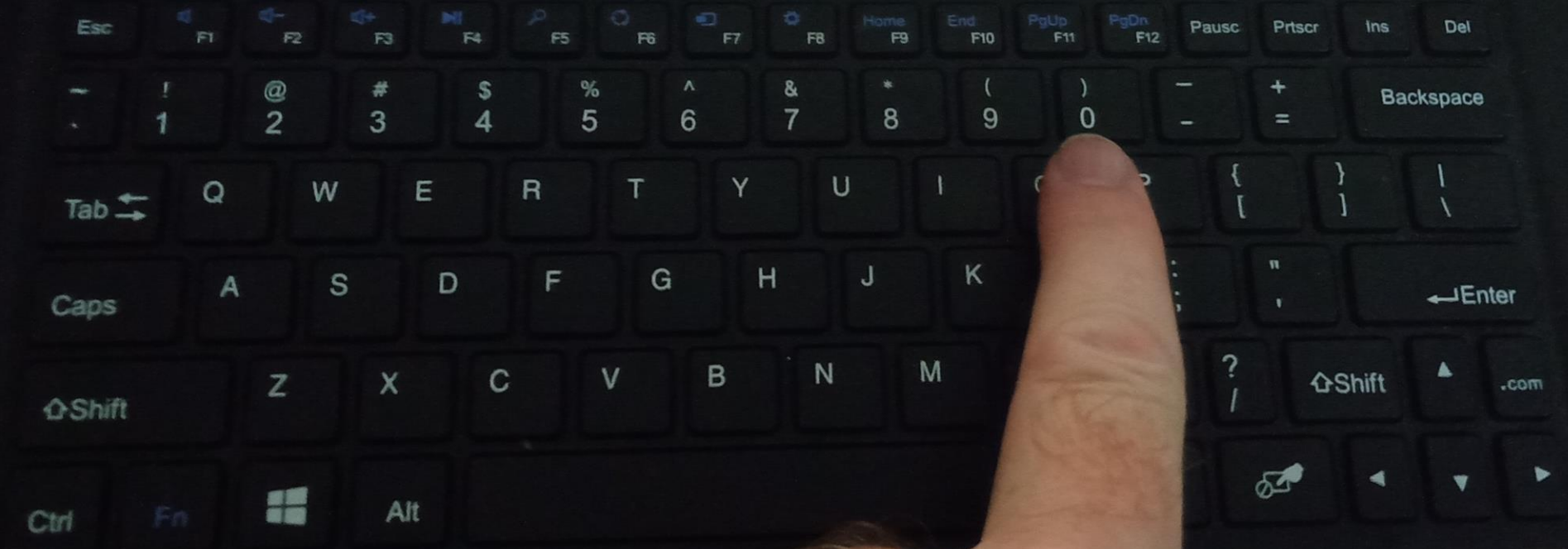


What **FACTS** do we Know?

Tap to add notes

Slide 1 of 227 English (Australia)

Notes





Tap to add notes

Slide 116 of 227



English (Australia)

Notes



O



Brahmagupta's 5 Addition Laws

AL1 positive plus positive is positive

AL2 negative plus negative is negative

AL3 positive plus negative is the difference between the positive and negative

AL4 when positive and negative are equal the sum is zero

AL5 positive plus zero is positive

negative plus zero is negative
zero plus zero is zero

Brahmagupta's 4 Multiplication Laws

ML1 The product of a negative and a positive is negative.

ML2 The product of two negatives is positive.

ML3 The product of two positives is positive.

ML4 The product of zero and a negative,
of zero and a positive, or
of two zeros is zero.

Brahmagupta's 5 Subtraction Laws

SL1 A smaller positive subtracted from a larger positive is positive.

SL2 A smaller negative subtracted from a larger negative is negative.

SL3 If a larger negative or positive is to be subtracted from a smaller negative or positive, the sign of their difference is reversed – negative becomes positive and positive negative.

SL4 A negative minus zero is negative,
a positive minus zero is positive,
zero minus zero is zero.

SL5 When a positive is to be subtracted from a negative or a negative from a positive, then it is to be added.

Brahmagupta's 4 Division Laws

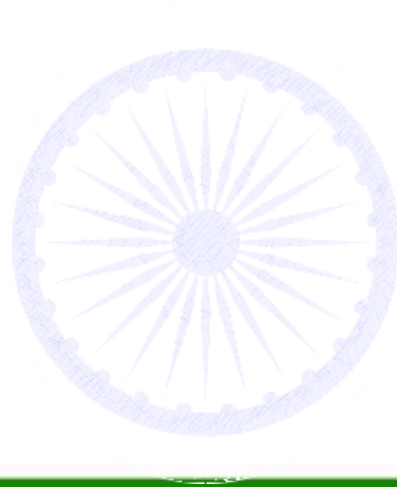
DL1 A positive divided by a positive is positive.

DL2 A negative divided by a negative is positive.

DL3 A positive divided by a negative is negative.

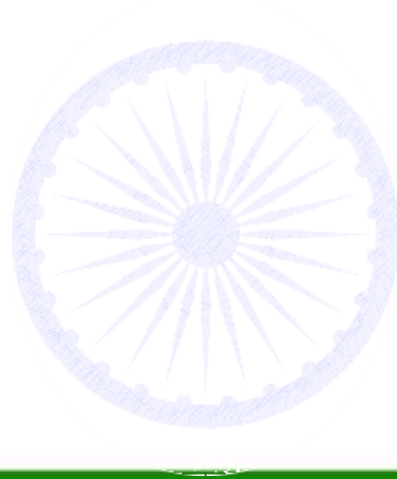
DL4 A negative divided by a positive is negative.

When **positive** and **negative** are equal the sum is **zero**



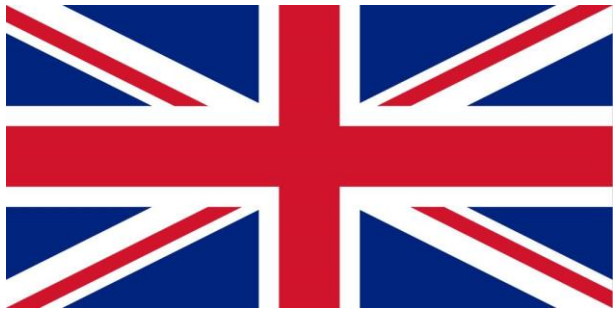
When **positive** and **negative** are equal the sum is **zero**

Correct according to the laws of physics,
yet **NOT** what we are taught in school!

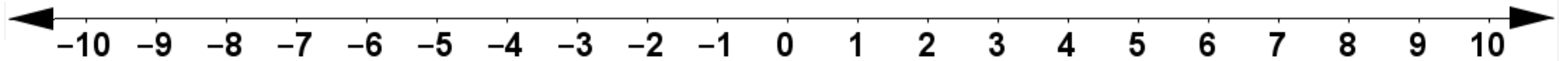


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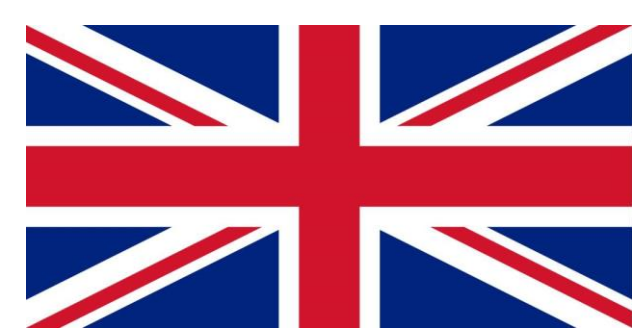
Negative Integers are less than Positive Integers.



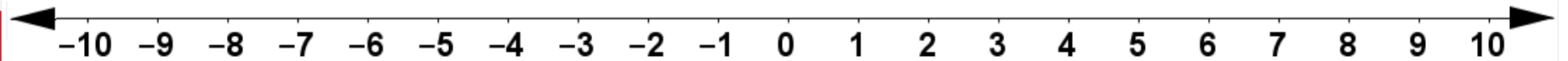
The more left you go, the smaller the number. NO!

When **positive** and **negative** are equal the sum is **zero**

Correct according to the laws of physics,
yet **NOT** what we are taught in school!



Negative Integers are less than Positive Integers.



The more left you go, the smaller the number. **NO!**

Maths pedagogies were developed on Naturals \mathbb{N}
then extended via subtraction to form Integers \mathbb{Z}

1 1 1 1 1 minus 1	+ 4 equals 1 1 1 1
1 1 1 1 1 minus 1 1	+ 3 equals 1 1 1
1 1 1 1 1 minus 1 1 1	+ 2 equals 1 1
1 1 1 1 1 minus 1 1 1 1	+ 1 equals 1
1 1 1 1 1 minus 1 1 1 1 1	0 equals

N

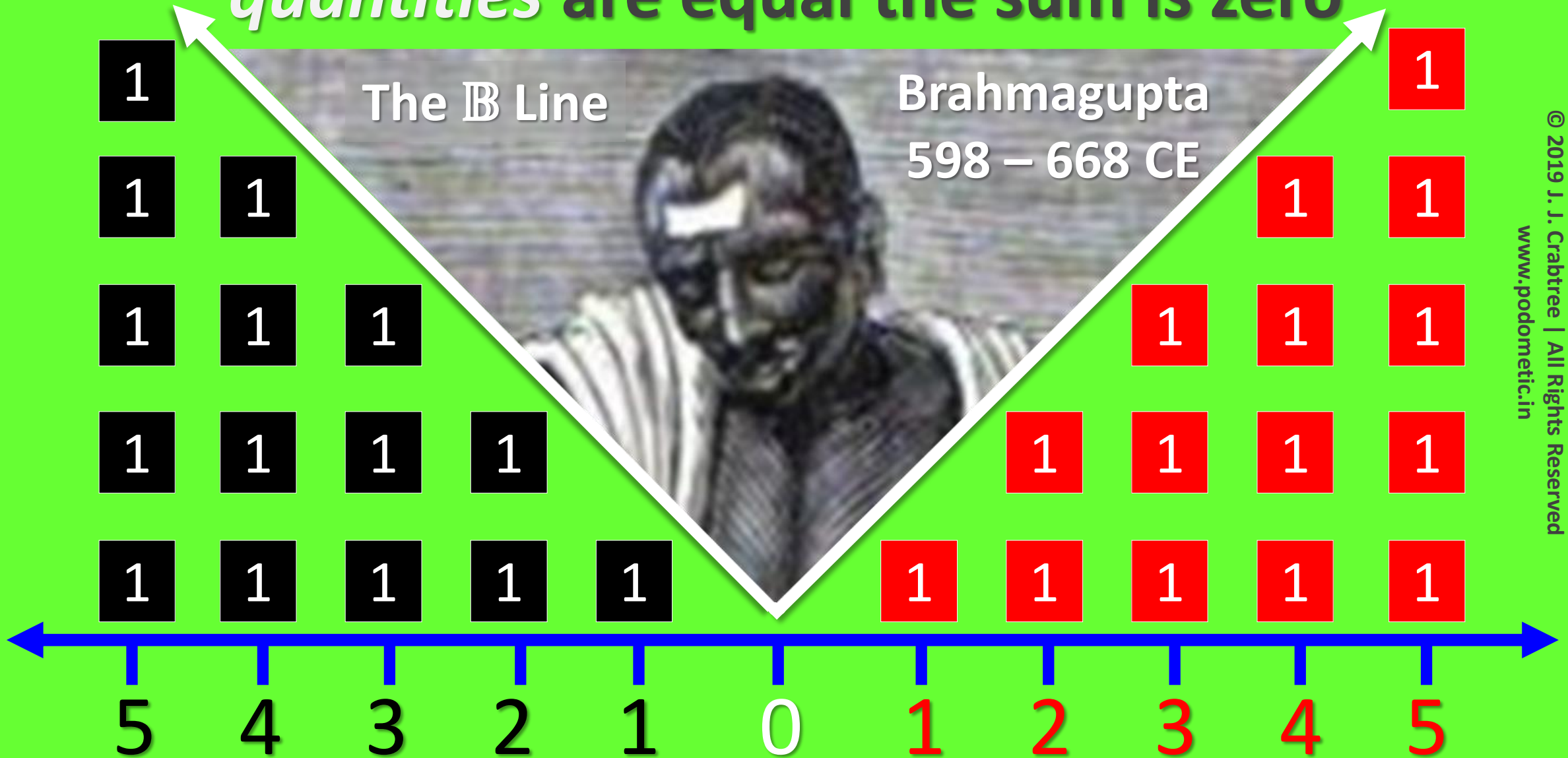
1 1 1 1 1 minus 1 1 1 1 1 1 - 1 equals 1

1 1 1 1 1 minus 1 1 1 1 1 1 1 - 2 equals 1 1

Z

The West wrongly extended the Naturals pattern onto the Integers.

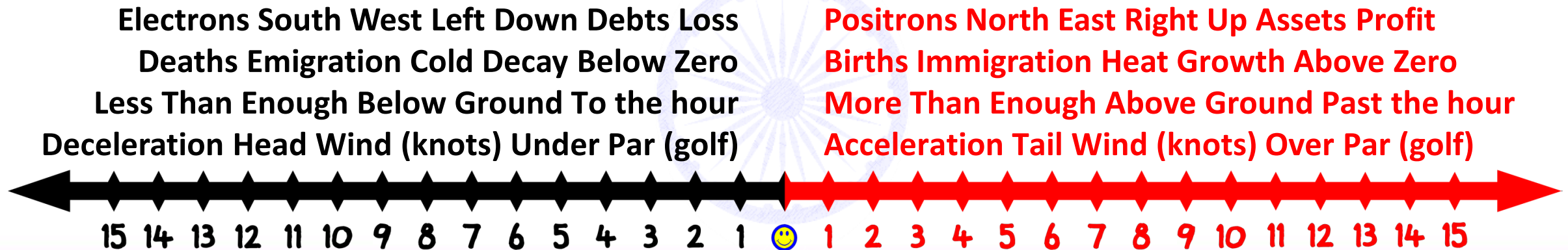
“when the number of positive and negative quantities are equal the sum is zero”



Seeing Simple Symmetries of Quantity

**Counts or measures
of negative units**

**Counts or measures
of positive units**



Brahmagupta's 5 Subtraction Laws

SL1 A smaller **positive** subtracted from a larger **positive** is **positive**.

$$+9 - +2 = +7$$

SL2 A smaller **negative** subtracted from a larger **negative** is **negative**.

SL3 If a larger **negative** or **positive** is to be subtracted from a smaller **negative** or **positive**, the sign of their difference is reversed – **negative** becomes **positive** and **positive** **negative**.

$$+4 - +6 = -2$$

SL4 A **negative** minus **zero** is **negative**,
a **positive** minus **zero** is **positive**,
zero minus **zero** is **zero**.

SL5 When a **positive** is to be subtracted from a **negative** or a **negative** from a **positive**, then it is to be added.

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Smaller
Negative!

$$-8 - (-5) = -3$$

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Smaller
Negative!

$$-8 - (-5) = -3$$

$$-3 - (-7) = +4$$

Larger
Negative!

Brahmagupta

SL1

A smaller

SL2

A small

SL3

If a large
negative
negative

SL4

A negative
a positive
zero

SL5

When a positive
or a negative from

Operation Laws

positive.

negative.

smaller

$$-8 - (-5) = -3$$

$$-3 - (-7) = +4$$

Smaller
Negative!

Larger
Negative!

Seeing Maths with Indian Eyes

Which numbers are **greater**?

 or 

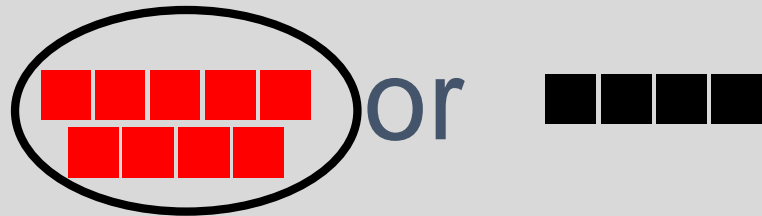
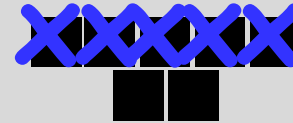
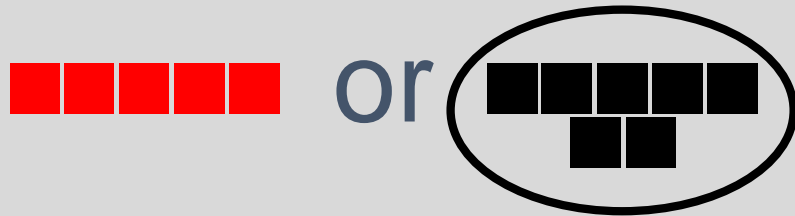
 or 

 or 

 or 

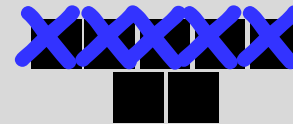
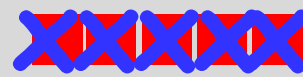
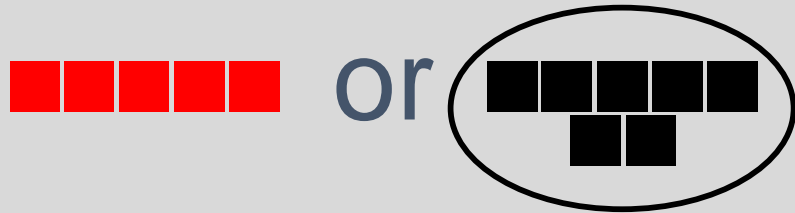
Seeing Maths with Indian Eyes

Which numbers are **greater**?

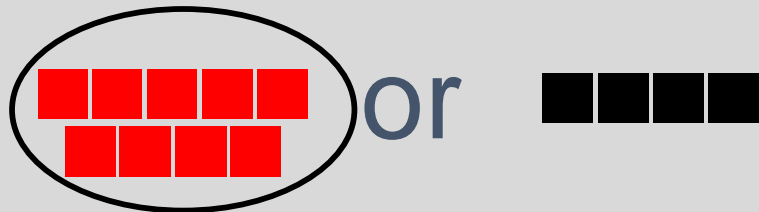


Seeing Maths with Indian Eyes

Which numbers are greater?



$$5^+ < 7^-$$



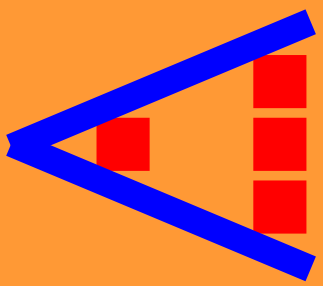
$$9^+ > 4^-$$



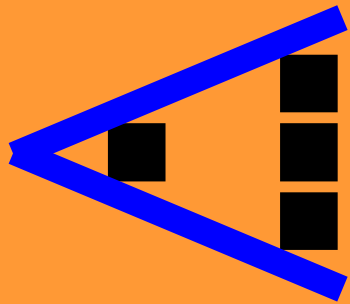
$$1^+ < 3^-$$



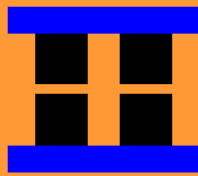
$$5^+ \neq 5^-$$



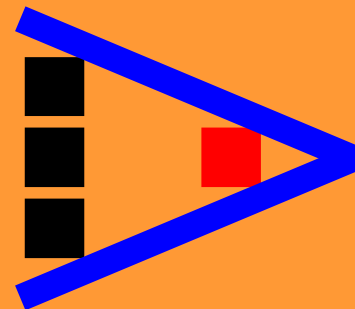
$$1^+ < 3^+$$



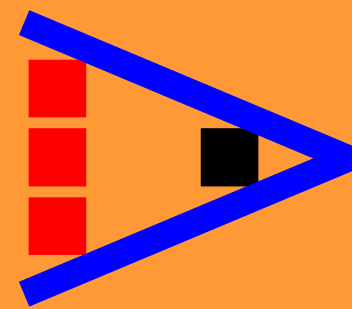
$$1^- < 3^-$$



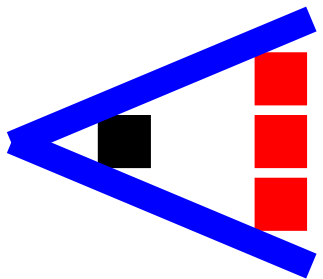
$$2^- = 2^-$$



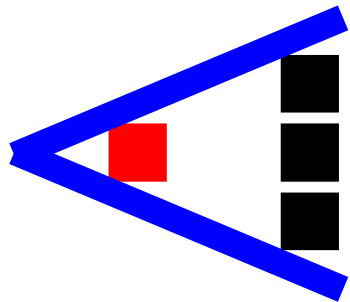
$$3^- > 1^+$$



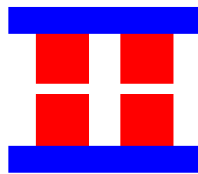
$$3^+ > 1^-$$



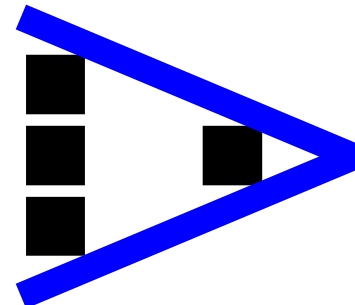
$$1^- < 3^+$$



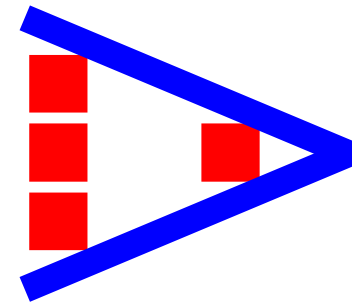
$$1^+ < 3^-$$



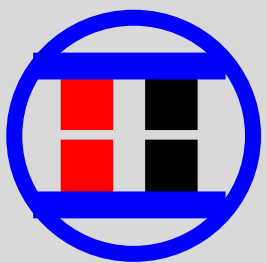
$$2^+ = 2^+$$



$$3^- > 1^-$$




$$3^+ > 1^+$$



2^+ are equal & opposite to 2^-


$$2^+ \ominus 2^-$$

JJC ASSESSMENT OF THE WORLD'S PEDAGOGICAL EVOLUTION (628 to Now)

$+12 + +4$	$+12 + -4$	$-12 + +4$	$-12 + -4$
$+12 - +4$	$+12 - -4$	$-12 - +4$	$-12 - -4$
$+12 \times +4$	$+12 \times -4$	$-12 \times +4$	-12×-4
$+12 \div +4$	$+12 \div -4$	$-12 \div +4$	$-12 \div -4$
	PASS	FAIL	Absent

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Podometric set to replace Arithmetic Dec. 2020

$+12 + +4$	$+12 + -4$	$-12 + +4$	$-12 + -4$
$+12 - +4$	$+12 - -4$	$-12 - +4$	$-12 - -4$
$+12 \times +4$	$+12 \times -4$	$-12 \times +4$	-12×-4
$+12 \div +4$	$+12 \div -4$	$-12 \div +4$	$-12 \div -4$
	PASS	FAIL	Absent

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Podometric set to replace Arithmetic Dec. 2020 with Free maths eBooks for every Indian child

$^{+}12 + ^{+}4$ $^{+}12 - ^{+}4$ $^{+}12 \times ^{+}4$ $^{+}12 \div ^{+}4$	$^{+}12 + ^{-}4$ $^{+}12 - ^{-}4$ $^{+}12 \times ^{-}4$ $^{+}12 \div ^{-}4$	$^{-}12 + ^{+}4$ $^{-}12 - ^{+}4$ $^{-}12 \times ^{+}4$ $^{-}12 \div ^{+}4$	$^{-}12 + ^{-}4$ $^{-}12 - ^{-}4$ $^{-}12 \times ^{-}4$ $^{-}12 \div ^{-}4$
	PASS	FAIL	Absent

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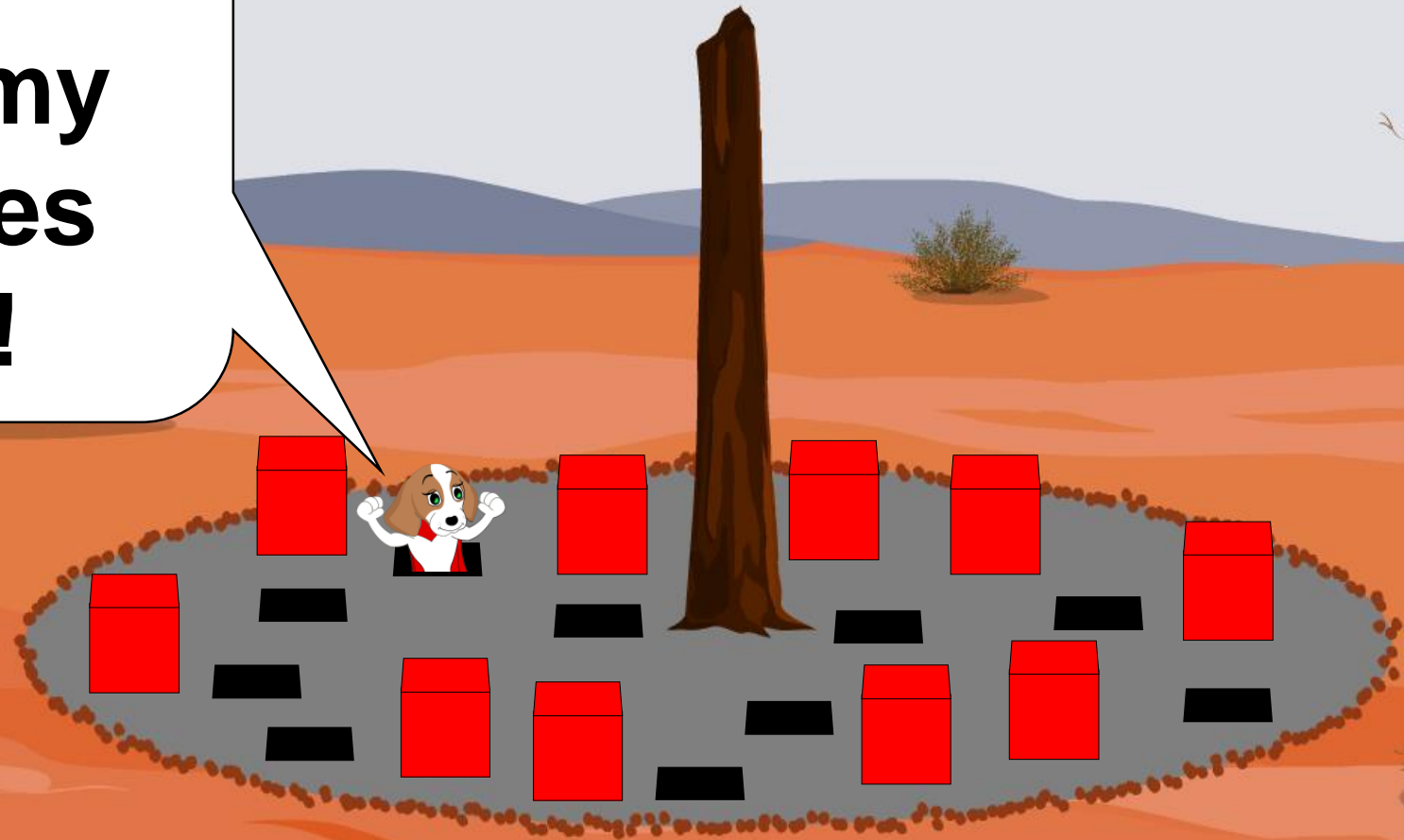
**Hey kids, play with me
and I'll make maths fun
fast and free to learn!**



Hey kids, play with me
and I'll make maths fun
fast and free to learn!
**From Class 1 on, it all
connects with the laws of
physics as well!**



**Separate, or
altogether, all my
bricks and holes
give me **zero**!**



I'm Podo the Puppy.
All my maths adventures
are being brought to life by
AFX Animation in Kolkata.
www.afxanimation.com



The hard work has been done!
India can update its
maths and prosper
or let this major maths
education advantage pass by.

**KEEP
DELHI
CLEAN**





Big Problems Demand We

Think bigger

Local Legends



ANDREW MATHIESON

BURIED among the hundreds of everyday emails in Jonathan Crabtree's inbox are a few worth keeping.

Replies from Nelson Mandela, Bishop Desmond Tutu, the Dalai Lama, even Muhammad Ali are quickly printed out and put aside for safe keeping.

They are a reminder of the power behind the written word; that ordinary people can make a difference.

"I haven't got one from the President of the United States or the Pope yet," Jonathan says, "but I'll get there soon."

"The emails are not really that important - it's much more about the ideas."



Brain power: Jonathan Crabtree has endless ideas for making the world a better place.

Picture: Tommy Ritchie 40442

truck trying to beat a red light 25 years ago nearly left him dead on the road after colliding with his motorcycle.

Lying motionless on the ground, Jonathan could only remember the horizon spinning around as if he were in a plane going down.

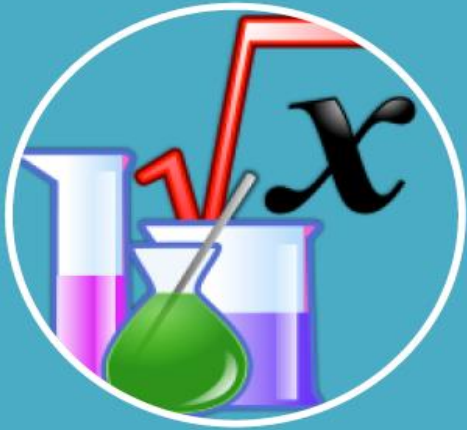
Consequently, the driver lost his licence but the

despite no formal qualifications, took up teaching mathematics from home.

"I actually failed maths," he laughs.

Throwing away the classroom text books, Jonathan taught kids to imagine algebra by closing their eyes and listening to fairytales.

Education authorities in Australia first shunned



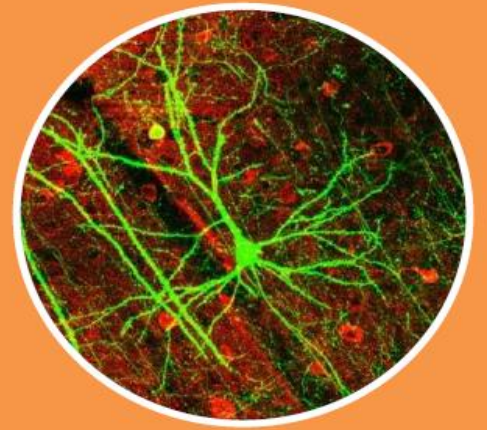
Science



Technology



Engineering



Medicine
In the West

MATHEMATICS

**India's teachers so seldom know
The trees of knowledge from seeds they sow
Past lives forgotten and the future a mystery
Making lives count, their deeds have made history**

A Seed to a View by Jonathan J. Crabtree

**So make your life count with love as your measure
Then kids will climb trees with views they will treasure.**

Thank you!

**If You Care
Please Share**

www.podometic.in

Thank You!

Please email me your feedback
feedback@podo.in



Jonathan J. Crabtree

Elementary Mathematics Historian, Melbourne, Australia

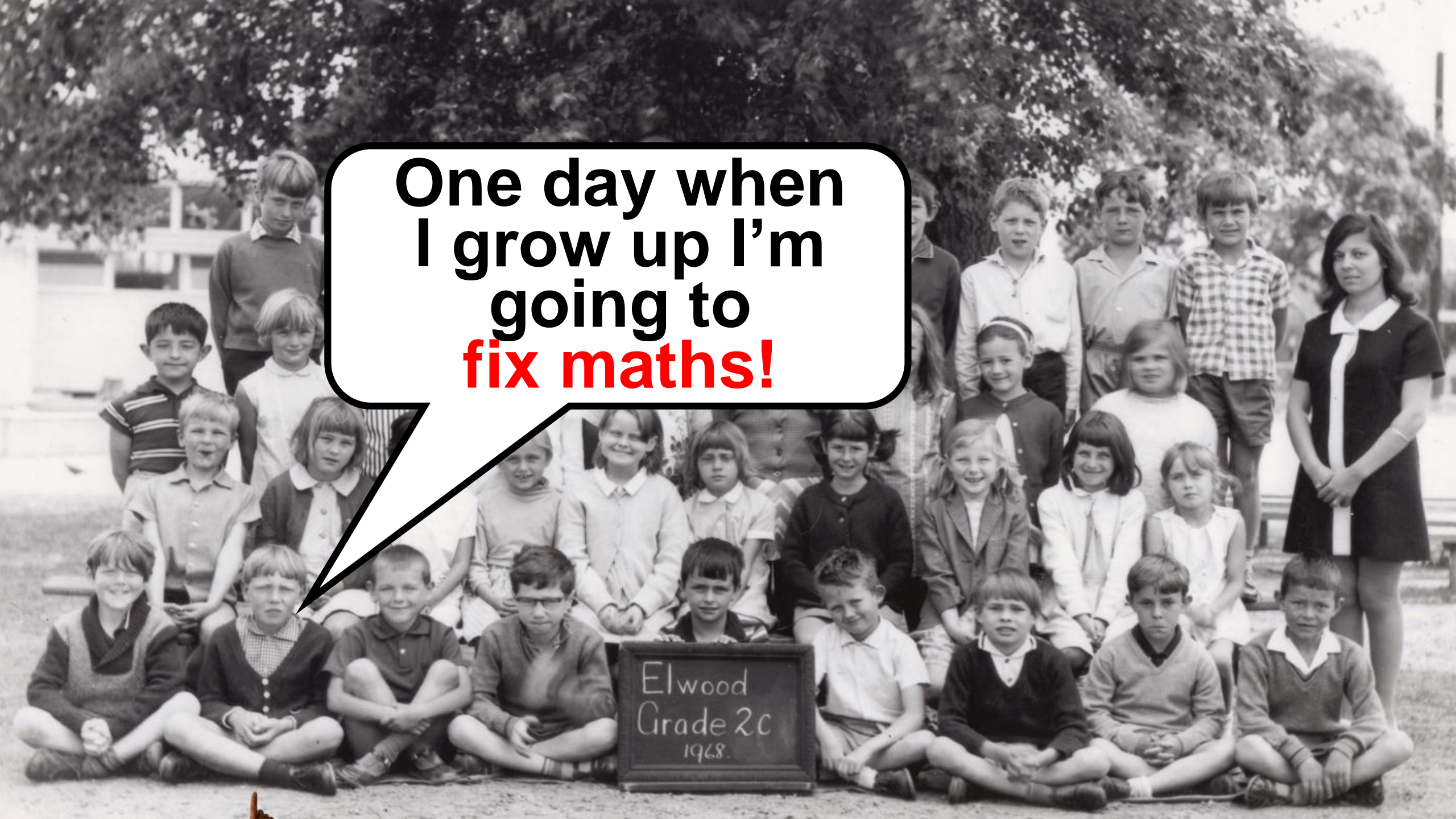
feedback@podo.in

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**I am Jonathan.
I am seven. I like
cartoons but,
I don't like maths!**

Elwood
Grade 2c
1968.



One day when
I grow up I'm
going to
fix maths!

Elwood
Grade 2c
1968.

PISA Mathematics Survey?

(Programme for International Student Assessment)

In Tamil Nadu and Himachal Pradesh 15% and 12% of students are ready to use mathematics in ways that are considered fundamental for their future development.

<https://www.acer.org/au/about-us/media/media-releases/acer-releases-results-of-pisa-2009-participant-economies>

PISA Mathematics Survey?

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The OECD average is 75%.

<https://www.acer.org/au/about-us/media/media-releases/acer-releases-results-of-pisa-2009-participant-economies>

PISA Science Survey?

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In Tamil Nadu and Himachal Pradesh 16% and 11% of students are proficient in science ... to participate actively in life situations related to science and technology.

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PISA Science Survey?

(Programme for International Student Assessment)

In **Tamil Nadu** and **Himachal Pradesh** **16%** and **11%** of students are proficient in science ... to participate actively in life situations related to science and technology.

The **OECD** average is **82%**.

<https://www.acer.org/au/about-us/media/media-releases/acer-releases-results-of-pisa-2009-participant-economies>

The English support the English cricket team

The English support the English cricket team

Indians support the Indian cricket team

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Indians support the Indian cricket team

The English support English mathematicians

The English support the English cricket team

Indians support the Indian cricket team

The English support English mathematicians

Indians support English mathematicians!

Mathematical Foundations?

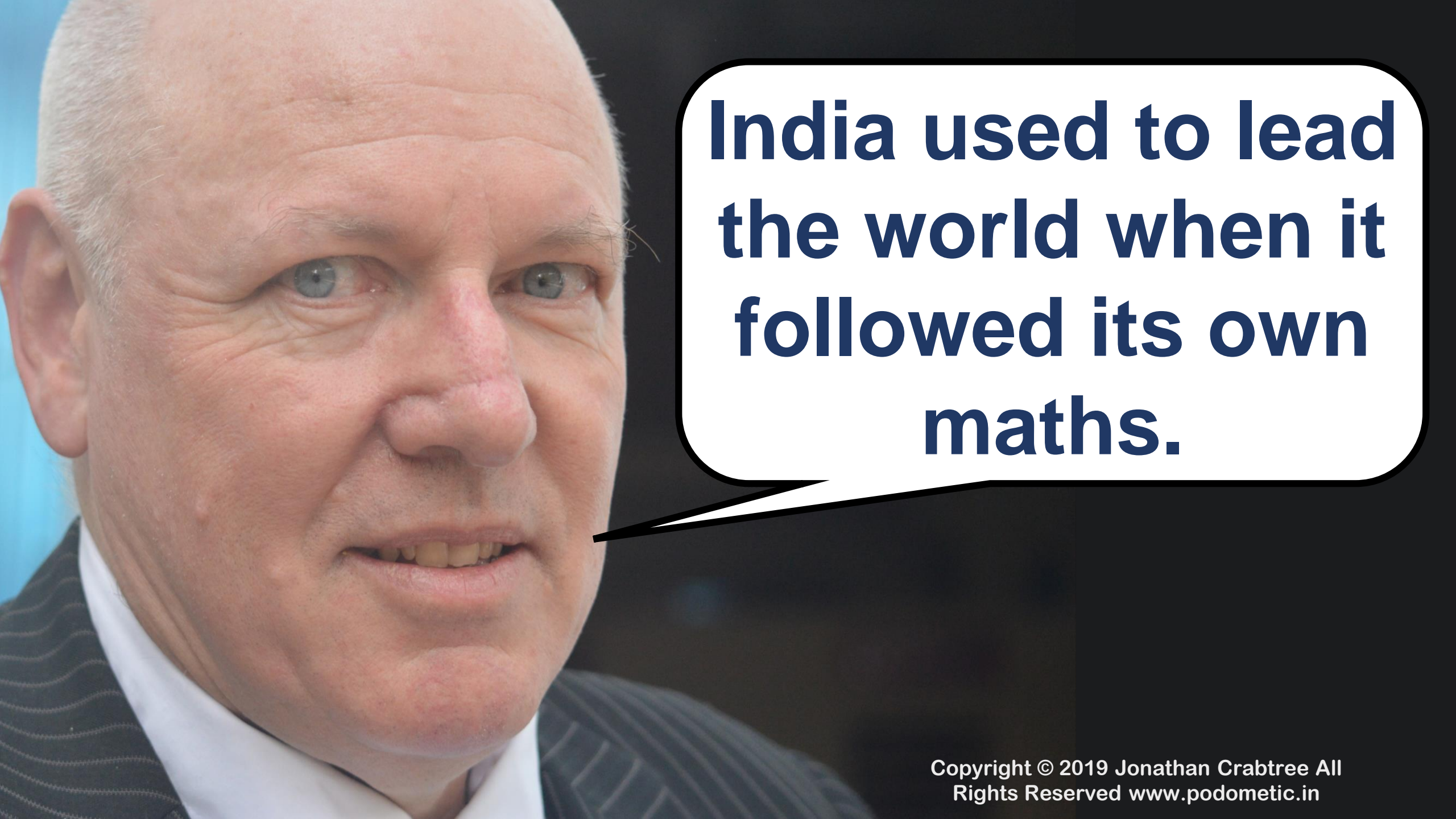


TRUE!

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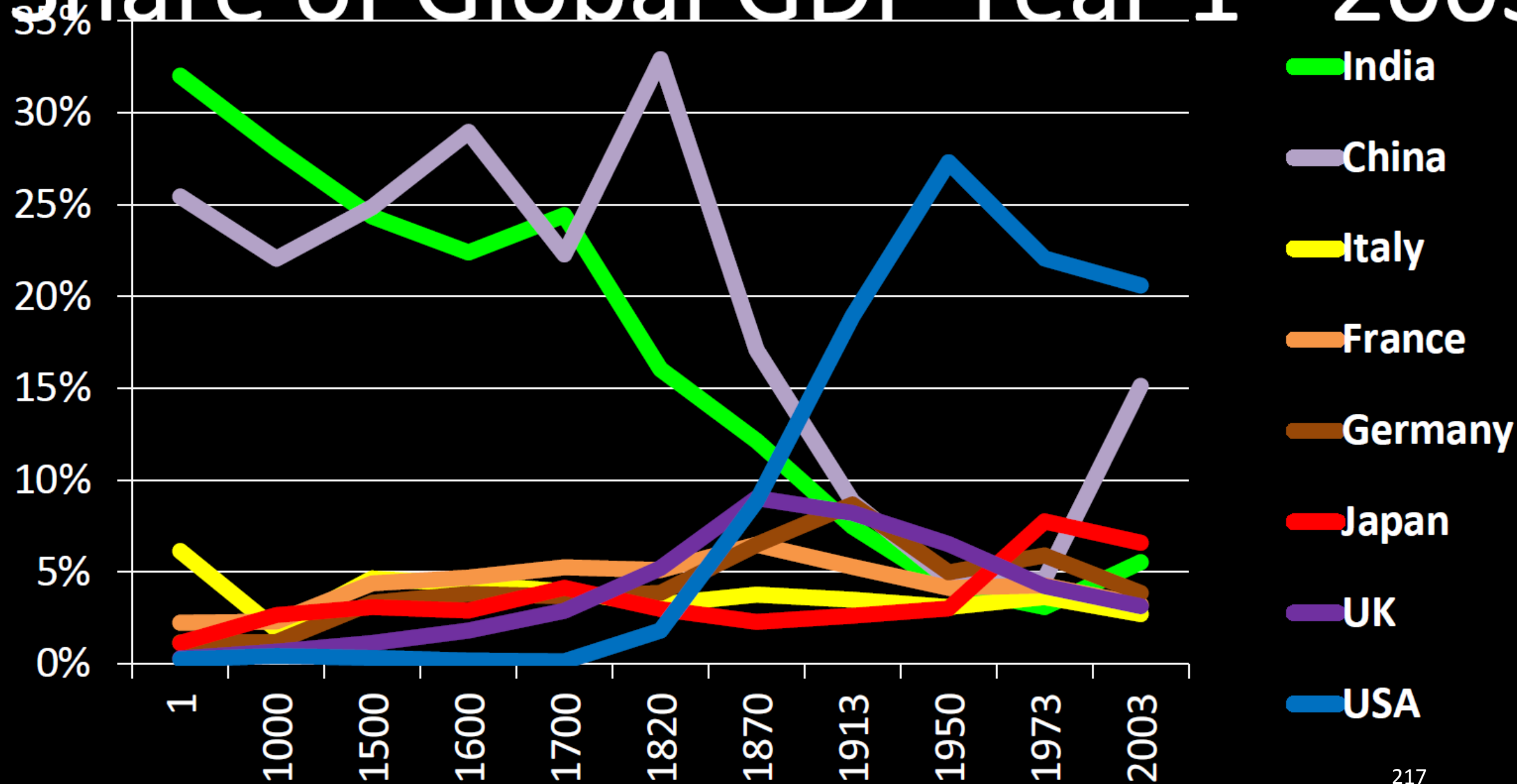


FALSE!



**India used to lead
the world when it
followed its own
maths.**

Share of Global GDP Year 1 - 2003



The hard work has been done!
India can update its
maths and prosper
or let this major maths
education advantage pass by.



**Thank you India for
your gift of zero. It
got lost, so I have
returned it to you.**



**If you care,
please share!**

Thank you.

Podometric is India's
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feedback@podo.in



Jonathan J. Crabtree

Elementary Mathematics Historian, Melbourne, Australia

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